

THE
TWO-REACTION THEORY
OF
SALIENT-POLE SYNCHRONOUS
MACHINES

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by

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C H A P T E R I

INTRODUCTORY

1. 1. SUMMARY

During the last twenty years many papers have been published, almost exclusively in the United States of America, on the two-reaction theory of salient-pole synchronous machines. These papers have ranged widely over aspects of the original idea from simple fundamentals to the detailed consideration of machine performance and the application to design.

Through the years papers have appeared giving basic contributions to the two-reaction concept, and many others have built elaborate amplifications with the intention of formalising methods of dealing with problems essentially very complex. As the literature grew it became evident that the variety of methods of approach and of basic assumptions had led to some confusion, making comparisons difficult and helping to obscure an already involved subject.

An attempt is made in the present work to survey the available material, to unify and combine the results obtained and to present a digest of the two-reaction theory in a more consistent form. So far as is known, no attempt of this kind has hitherto been made, at any rate in Britain. It is therefore the main purpose of the present thesis to furnish a critical review of the subject at its present stage, when the basic principle is still recognisable and elaborations are not so great as to overweight its usefulness.

1. 2. ORIGINS

The two-reaction method of analysis was first suggested by André Blondel (12). Doherty and Nickle (14) extended the idea in 1926, and as a consequence of the wider publicity, stimulated other engineers to an exhaustive study of the theory. Wiese man (22) determined flux distribution coefficients used for the calculation of reactances graphically by flux plotting for a variety of shapes of poles, while Doherty and Nickle (12) derived reactance formulae analytically from permeance curves. For the determination of rotor-circuit reactances Linville (30) introduced the equivalent circuit, which was developed into a complete form by Rankin (54). To enable the machine constants to be calculated from design data, accurate formulae for leakage, synchronous, transient, sub-transient and negative-phase-sequence reactances and time-constants were derived in many papers by Alger, Kilgore, Linville and Rankin. Different test methods for the determination of the machine constants were also suggested by various authors, including Wright, Tracy and Tice, and others.

(12) For numbered reference, see Bibliography.

As to the characteristics and performance of motors and generators expressed in terms of machine constants, Doherty and Nickle (23, 28) derived detailed expressions for the treatment of single-phase and three-phase short circuits based on the "constant flux-linkage" theorem. Putman, Linville, and Liwschitz, derived formulae for starting performance. Transient torque-angle characteristics, short-circuit torque, and nearly all other machine performances were studied and formulae derived by various writers.

While most of the investigators studied the theory from a physical point of view, Park (27, 41) analysed it on a purely mathematical basis using Heaviside's Operational Calculus, and derived general formulae for nearly all the aspects of machine performance. Following this fundamental idea, the method was extended by other authors to the analysis of machines with more complicated armature circuits.

The "Per-unit value" system was used by most authors, but various base rotor currents and therefore effective turn-ratios were severally employed. Rankin (52, 53) gave a clear comparative discussion of "bases" and derived conversion formulae by which any ampere-length quantity could be converted to a per-unit value for any base rotor current.

Thus the two-reaction theory has developed from a primitive idea to a more or less complete, comprehensive and elaborate form.

1. 3. SCHEME

The original concept of Blondel and its simpler implications are outlined in Chapter II. The armature m.m.f., assumed to be sinusoidally distributed, is considered as composed of direct- and cross-reaction components, each similarly sinusoidal. A discussion of the actual flux wave form, with its fundamental and harmonic content as derived graphically and analytically, follows in Chapter III.

In Chapter IV are a detailed discussion of all the machine constants, and also Park's flux linkages formulae, which are the bases for analysis. Chapter V gives the per-unit system, a discussion of the different bases, and the preferred base employed in this thesis. In Chapter VI are the flux linkages and voltages equations of all the armature and rotor circuits, wherefrom the operational impedances are derived and the equivalent circuits are formed.

The calculation of coefficients is discussed in Chapter VII

followed by their determination by test in Chapter VIII. Chapter IX gives an outline of the characteristics and performance of a machine expressed in terms of its measured or calculated constants. The remainder of the work is concerned with the application of the two-reaction theorem in design.

1. 4. BASIC ASSUMPTIONS

While a variety of assumptions is made in dealing with different aspects of the theory, some of the assumptions are common and basic, and apply to every aspect. They are:-

- A. There is no magnetic saturation, and hysteresis effects are absent.
- B. The magneto-motive force of the three-phase armature currents is, under normal working conditions, distributed as a sine wave along the airgap periphery, one complete sinusoid corresponding to the length of one double pole-pitch.

Saturation is, of course, unavoidable in any practicable machine, and account is taken of it by empirical modifications. Any attempt to take account of m.m.f. harmonics is extremely burdensome, and in any case unjustified.

CHAPTER II

THE BASIC TWO-REACTION THEOREM

2. 1. FUNDAMENTAL IDEAS

Because of the pole-shape in salient-pole machines, the magnetic permeance presented to the armature-reaction m.m.f. is not constant along the gap periphery. It is greatest over the pole faces and a minimum in the direction of the inter-polar axis of the field system. Since the field structure is constructionally symmetrical about the polar and interpolar axes, it is possible and convenient to take the variation into account by first resolving the armature reaction m.m.f. into two components, one in the direction of each of these axes.

Fig. 2.1 shows the armature-reaction m.m.f. wave form of a salient-pole machine, assuming that the total armature m.m.f. wave is sinusoidally distributed with amplitude A along the airgap. This wave, under steady condition, moves in synchronism with the field system and may be resolved into two sinusoidal waves in space-quadrature, each likewise moving in synchronism with the field. The two component waves are designated A_d and A_q . A_d is the direct-axis m.m.f. component, with its crest on the same axis as the main poles, and with either an aiding or an opposing effect on the field's own m.m.f. A_q is the quadrature-axis m.m.f. component and has a "cross-magnetizing" effect on the field as it strengthens the existing field m.m.f. at one pole tip and weakens it at the other.

Owing to the complicated manner in which the permeance to these armature reaction m.m.f.'s., varies along the airgap, the flux-density space waves will be symmetrical but of complicated shape in an actual machine. Neglecting saturation, the space distribution of the flux density can be found by combining the m.m.f. waves with an appropriate distributed permeance wave, as suggested by Doherty and Nickle (14).

The flux wave in each axis, Fig. 2.2, produced by the appropriate components of a sine-distributed m.m.f., A , will then have a shape analysable into a fundamental and harmonics, all moving at synchronous speed. The fundamental flux wave in each axis will generate in the armature winding a fundamental-frequency induced e.m.f., while the space harmonics of flux will give rise to harmonic e.m.f.'s.

The fundamental e.m.f.'s., in the two axes do not bear the same proportion to each other as do A_d and A_q , the direct- and quadrature-axis m.m.f., components, because of the great difference in the effective permeances presented to them by the gap.

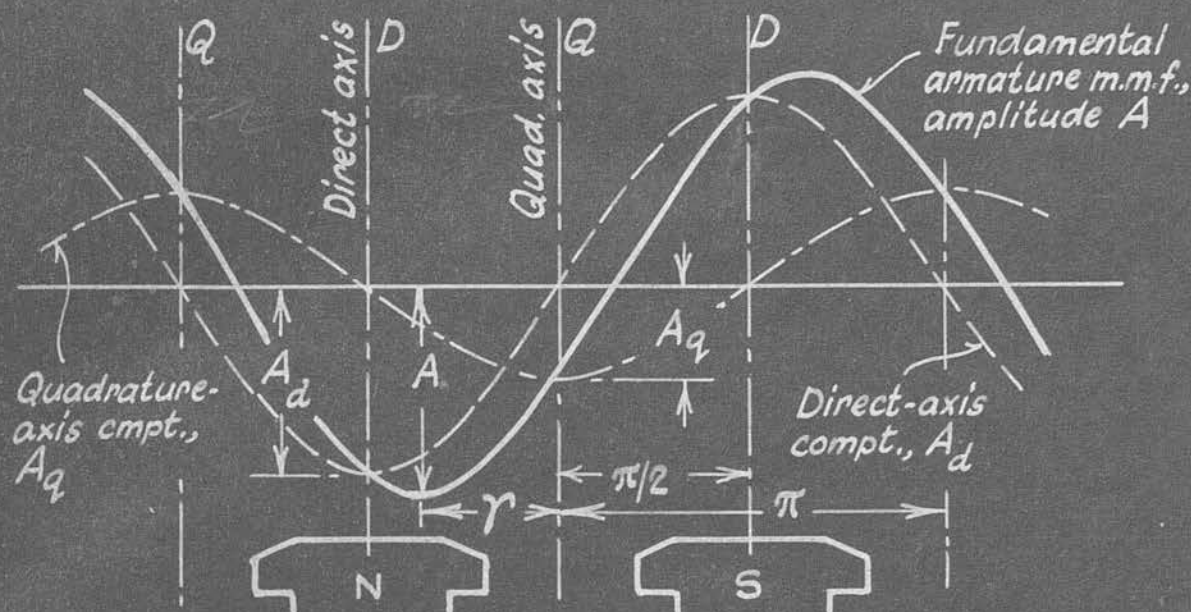


FIG.2.1. Armature-reaction m.m.f. waves of salient-pole generator.

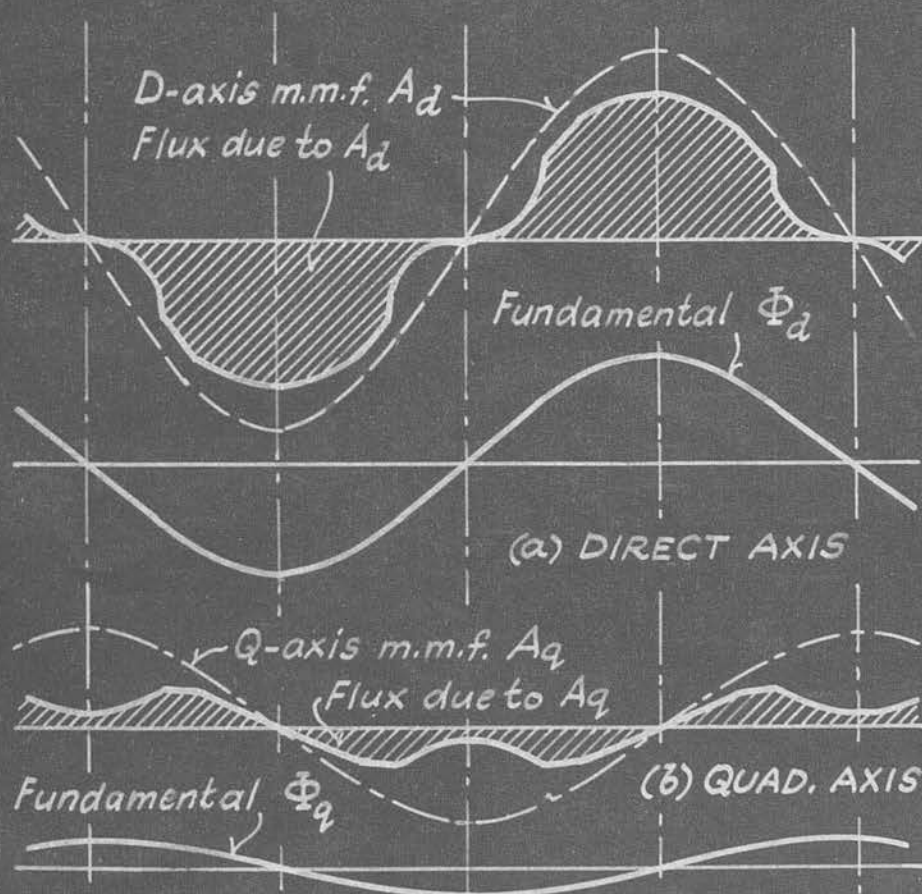


FIG.2.2. Armature-reaction flux-wave components.

2. 2. VECTOR DIAGRAM

If now the flux harmonics are neglected, the remaining fundamental fluxes may be represented by space vectors. A_d and A_q can also - and more legitimately - be represented by space vectors, while the fundamental voltages and currents can be represented by time vectors. In order to show the relations between the currents, m.m.f.'s., fluxes and voltages the space and time vectors are combined in one diagram as shown in Fig. 2.3. This diagram is a combined space and time vector diagram of a salient-pole alternator with a lagging current, under steady-state conditions.

A is the armature reaction m.m.f. which is in phase with the armature current i : it is resolved into A_d and A_q in the direct and quadrature axes. A_d and A_q produce fundamental fluxes Φ_d and Φ_q in the direct and quadrature axes respectively: Φ_d and Φ_q are not in the same proportion as A_d and A_q because of the different permeance coefficients in the two axes, and therefore the resultant flux Φ_a of Φ_d and Φ_q is not co-phasal with A . Φ_a and the field flux Φ_f combine together to form the resultant flux Φ which produces the airgap voltage e lagging 90° on Φ . If there were no armature reaction, the field flux Φ_f alone would then produce the nominal open-circuit voltage e_n . Therefore e is equal to the vector sum of e_n and the reactive voltages e_{Φ_d} and e_{Φ_q} produced by Φ_d and Φ_q respectively. $X_l i$ is the leakage reactance drop and $r_a i$ is the resistance drop. The airgap e.m.f. e less the resistance drop $r_a i$ and leakage reactance drop $X_l i$ gives the terminal voltage e_x .

It is to be noted that the pole axis, and not the terminal phase angle, determines the resolution of A into its direct- and quadrature-axis components A_d and A_q .

2. 3. ARMATURE-REACTION REACTANCES

Consider a non-salient-pole machine. The flux physically existing is a result of the combination of field and armature m.m.f. but if saturation is neglected the effect is the same as the combination of the separate fluxes produced by the m.m.f.'s., acting separately, obtained by any desired subdivision of the total m.m.f. Again, the total e.m.f. can be considered as the summation of the e.m.f.'s., generated by these separate fluxes.

Because the flux component Φ_a due to armature m.m.f. is proportional to the current, the e.m.f. generated by it, e_{Φ_a} , is also proportional to the current, and might instead be expressed in magnitude as equal to the current multiplied by a reactance coefficient X_a , so that $e_{\Phi_a} = i X_a$.

For the salient-pole machine a similar convention may be adopted. Doherty and Nickle (14) accordingly resolve the armature

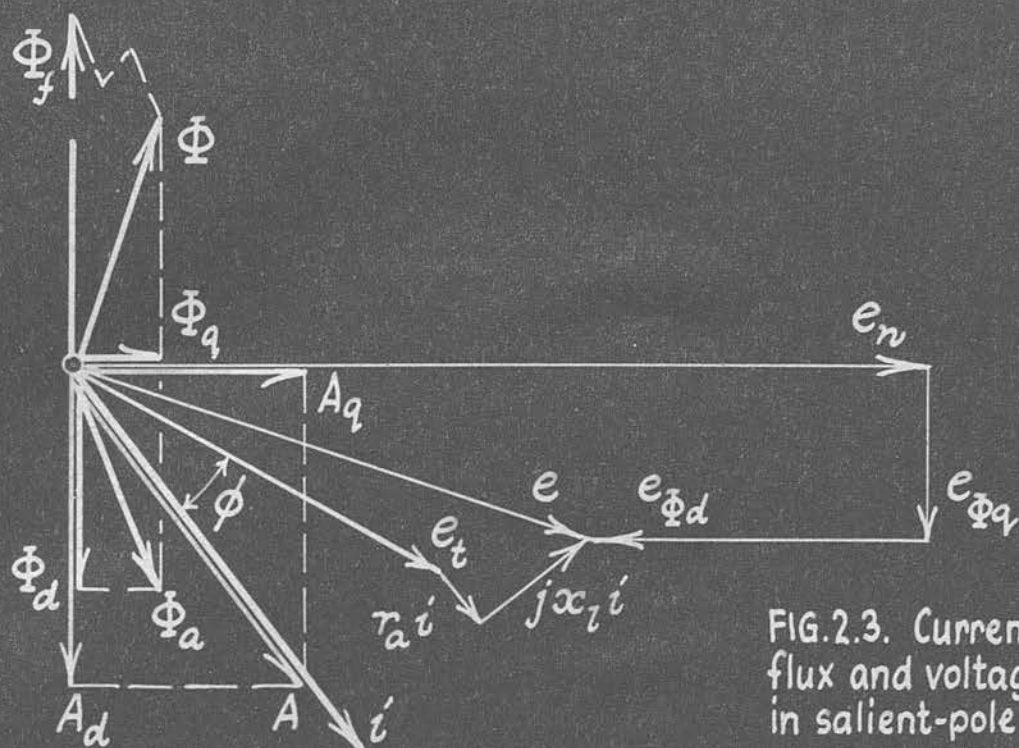


FIG. 2.3. Current, m.m.f., flux and voltage vectors in salient-pole generator.

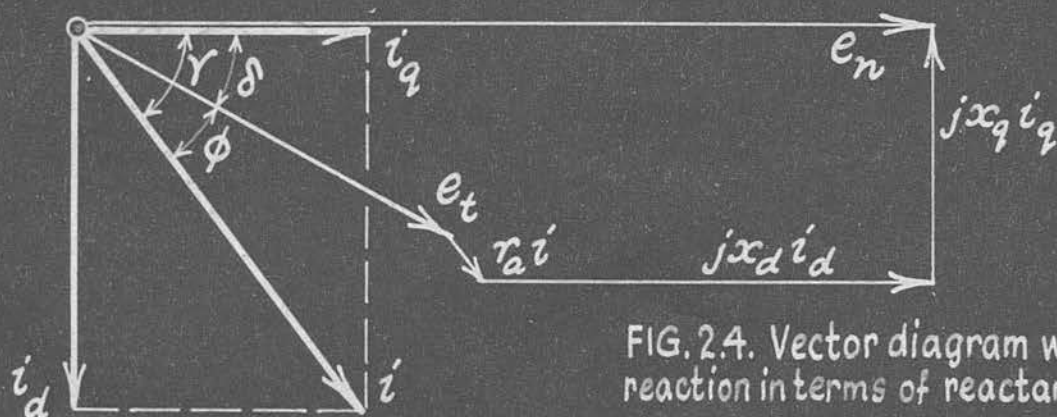


FIG. 2.4. Vector diagram with reaction in terms of reactances.

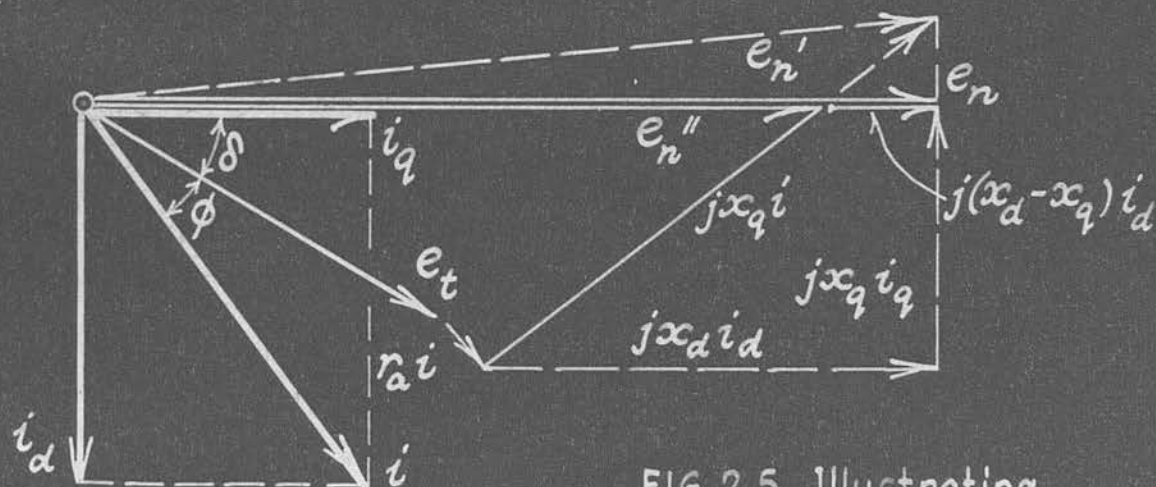


FIG. 2.5. Illustrating method of construction.

current i into two mutually perpendicular components i_d and i_q in the direct and quadrature axes respectively. The m.m.f's., A , A_d and A_q are indeed proportional to the currents i , i_d and i_q . With the component i_d there is then associated a direct-axis armature-reaction reactance X_{ad} , and to i_q a quadrature-axis armature-reaction reactance X_{aq} . X_{ad} is therefore the ratio of the fundamental reactive voltage $i_d X_{ad}$ produced by the direct-axis component i_d of a fundamental current to that component of current. X_{aq} is similarly defined as the ratio of the fundamental reactive voltage produced by the quadrature-axis component of a fundamental current to that component of current. Thus X_{ad} and X_{aq} will give rise to reactance voltage drops, $j X_{ad} i_d$ and $j X_{aq} i_q$ in leading quadrature with i_d and i_q respectively. The armature reaction effects consequently become now represented by two armature-reaction-reactance voltage drops.

As in a cylindrical rotor machine the armature-reaction reactance together with the armature leakage reactance sum to the synchronous reactance of the machine, so the synchronous reactance of a salient-pole machine has two values: the direct-axis synchronous reactance X_d and the quadrature-axis synchronous reactance X_q . If the armature leakage reactance X_l is taken to be same in both axes, then

$$X_d = X_l + X_{ad}$$

$$X_q = X_l + X_{aq}$$

If the saturation is neglected, X_d and X_q are constant. The vector diagram of a salient-pole alternator with lagging current is then as shown in Fig. 2.4, drawn for the same operating conditions as Fig. 2.3.

It will be seen that the air-gap e.m.f., the armature m.m.f's. and their component fluxes are omitted, and that a direct vector relation is obtained between the terminal voltage E_t , the nominal open-circuit e.m.f. E_n , and the reactance voltages $X_d i_d$ and $X_q i_q$, based on the resolution of i along and in quadrature with the pole axis.

2. 4. GEOMETRY OF THE REACTANCE VECTOR DIAGRAM

Let E_t = terminal voltage,

i = load current,

ϕ = terminal phase-angle,

r_a = armature resistance,

X_d, X_q = synchronous reactances of the direct and quadrature axes;

then the no-load e.m.f. E_n and the power angle δ can be found from the six above quantities.

2. 4. (1) ANALYTICALLY

From Fig. 2.4

$$\begin{aligned} E_n &= E_t \cos \delta + r_a i \cos \phi + X_d i_d \\ &= E_t \cos \delta + r_a i \cos (\phi + \delta) + X_d i \sin (\phi + \delta) \end{aligned} \quad (2.1)$$

$$E_t \sin \delta + r_a i \sin (\phi + \delta) = X_g i_g = X_g i \cos (\phi + \delta)$$

$$\begin{aligned} \text{Or } E_t \sin \delta + r_a i \sin \phi \cos \delta + r_a i \cos \phi \sin \delta \\ = X_g i \cos \phi \cos \delta - X_g i \sin \phi \sin \delta \end{aligned} \quad (2.2)$$

Solving (2.2) for $\tan \delta$,

$$\tan \delta = \frac{X_g i \cos \phi - r_a i \sin \phi}{E_t + r_a i \cos \phi + X_g i \sin \phi} \quad (2.3)$$

So δ can be obtained from the known quantities. By substituting the value of δ into (2.1), E_n can also be obtained.

2. 4. (2) GRAPHICALLY

From Fig. 2.4

$$\begin{aligned} E_n &= E_t + r_a i + j X_d i_d + j X_g i_g \\ &= E_t + r_a i + j X_g (i_g + i_d) + j (X_d - X_g) i_d \\ &= E_t + r_a i + j X_g i + j (X_d - X_g) i_d \end{aligned}$$

$$\text{Let } E_n'' = E_t + r_a i + j X_g i$$

$$\text{Then } E_n = E_n'' + j (X_d - X_g) i_d \quad (2.4)$$

By definition of i_d , it is in quadrature with E_n . Hence $j (X_d - X_g) i_d$ is cophasal with E_n .

From (2.4), because E_n is the vector sum of E_n'' and $j (X_d - X_g) i_d$, E_n'' has the same direction as both these voltages, and the magnitude of E_n is the scalar sum of E_n'' and $(X_d - X_g) i_d$.

Thus to find E_n : first add the resistance drop $r_a i$ in appropriate phase to the terminal voltage E_t . Then consider the machine to have a reactance X_g and add a reactance drop $X_g i$ in leading quadrature with i to locate the direction and the

magnitude of the fictitious e.m.f. E_u'' . Finally extend E_u'' to E_u by the length $(X_d - X_g) i_d$. The angle included between E_u and E_x is δ .

The graphical method of determining E_u and δ is shown in Fig. 2.5.

CHAPTER III

FLUX DISTRIBUTION

3. 1. FLUX DISTRIBUTIONS IN A SALIENT-POLE MACHINE

The distribution at the armature surface of the flux of a salient pole is a function of the pole-arc/pole-pitch ratio, the gap length and the pole-shoe profile. The method of developing the shape of the distribution makes it convenient to assume that the m.m.f. of the pole is a square wave centred on the pole-axis (i.e. the direct axis).

As already discussed, the armature m.m.f. is assumed to be sinusoidal, and to be resolved into sine components centred on the direct and quadrature axes.

Fig. 3.1 shows the assumed m.m.f. square wave for an excited salient pole, together with a typical shape of the flux distribution due to it at the armature gap-surface. The flux can be analysed into a space fundamental and harmonics. The fundamental flux wave is responsible for the fundamental no-load "excitation voltage", while the harmonics produce harmonic voltages. The harmonic voltages are usually very small in comparison with the fundamental voltage and they are usually neglected.

When a salient-pole machine is excited only by a sine-wave armature reaction m.m.f. wave in the direct-axis, the flux wave is more pointed than that due to the field alone, and is as roughly shown in Fig. 3.2.

This flux wave also consists of a fundamental and harmonics. The fundamental produces the direct-axis armature-reaction fundamental reactive voltage, while the harmonics produce reactive harmonic voltages which are here neglected.

When a salient-pole machine is excited only by a sine-wave armature reaction m.m.f. in the quadrature-axis, the flux wave is saddle-shaped, Fig. 3.3.

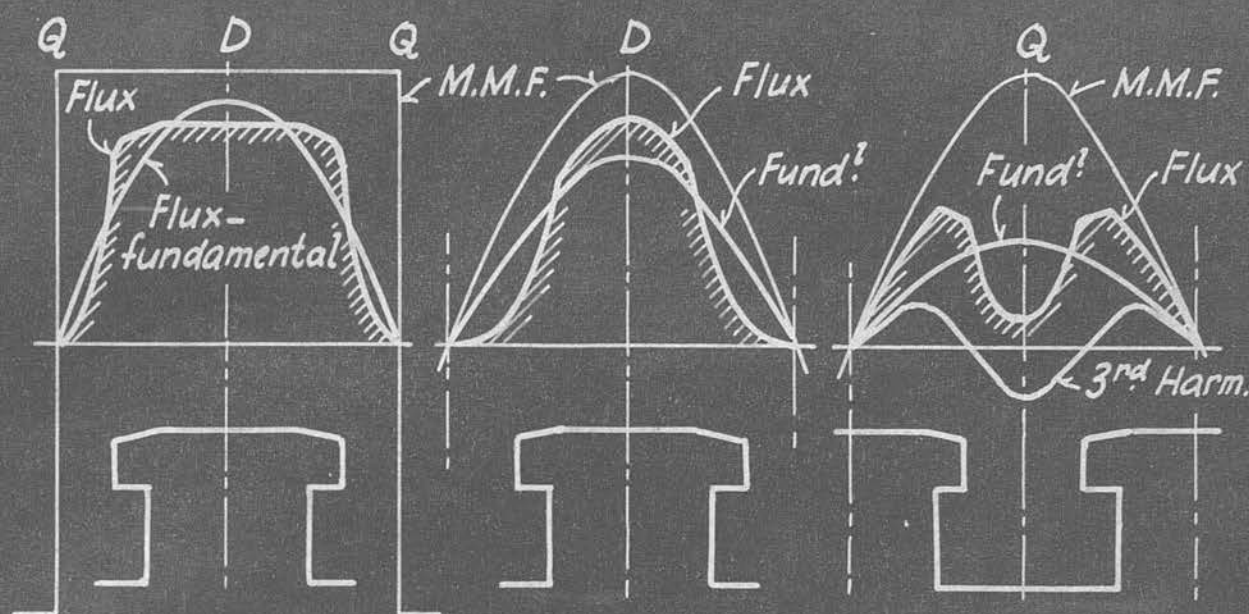
Again, the flux wave may be resolved into a fundamental and harmonics. It is apparent from the figure that the fundamental flux is much smaller than that in the direct-axis for the same m.m.f., while there is a very prominent third harmonic due to the peculiar distribution of the permeance.

3. 2. FLUX DISTRIBUTION COEFFICIENTS

See Fig. 3.4. Let

F = field winding ampere-turns per pole for the airgap at no-load and rated voltage

$$= N_{fd} I_{fd}$$



Flux distributions in salient-pole machine

FIG.3.1. Excited by field winding only.

FIG.3.2. Excited by sine armature m.m.f. in D-axis.

FIG.3.3. Excited by sine armature m.m.f. in Q-axis.

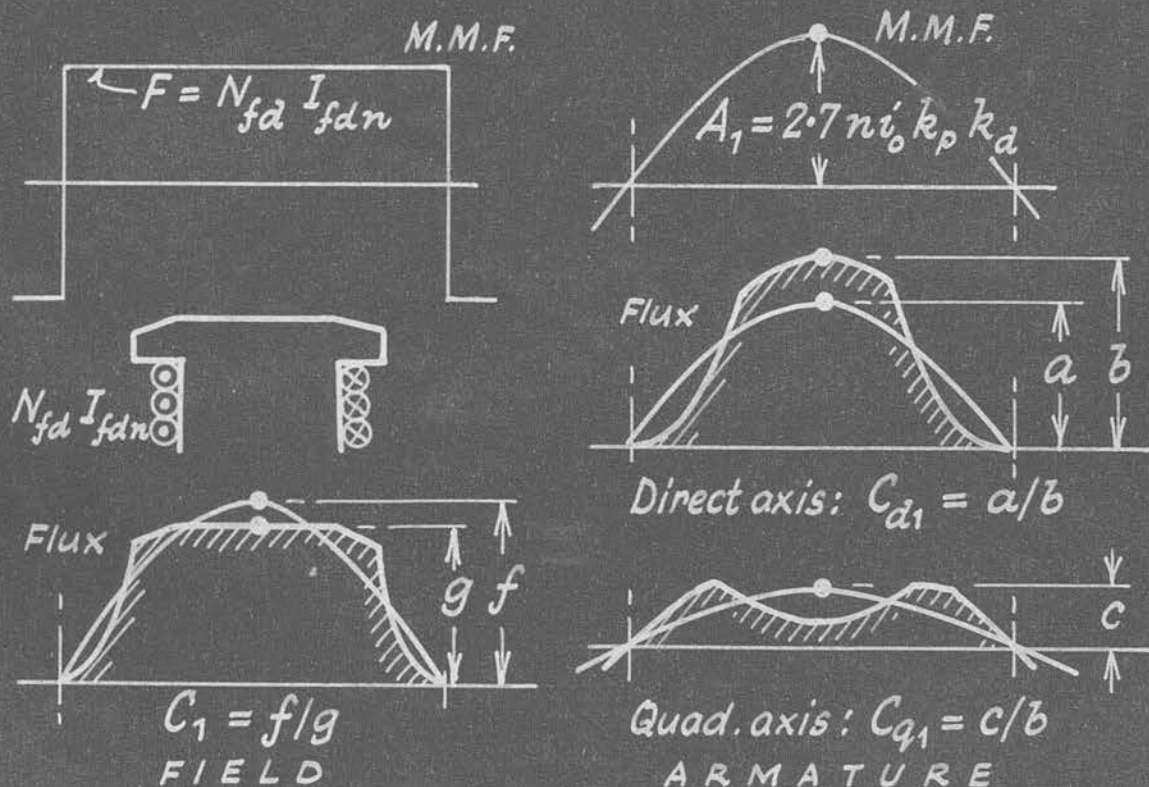


FIG.3.4. Flux-distribution coefficients.

where N_{fd} = field turns per pole
 and I_{fdn} = field current at no-load and rated voltage

A_1 = maximum fundamental armature ampere-turns

$$= \frac{3}{2} \cdot n \cdot k_p k_d \cdot \sqrt{2} i_o \cdot \frac{4}{\pi} = 2.7 n i_o k_p k_d$$

where n = armature turns per phase per pole

k_p = pitch factor

k_d = breadth factor

and i_o = r.m.s. rated armature current.

A_{1f} = demagnetizing ampere-turns.

= field ampere-turns required to balance armature reaction (on the basis that the fundamental of the field flux wave balances the fundamental of the armature flux wave) at rated current and at zero power factor.

C_1 = ratio of fundamental airgap flux by the field to the actual maximum value at the pole-centre.

C_{d1} = ratio of fundamental airgap flux by a sine-wave armature m.m.f. in the d-axis to the maximum value.

C_{q1} = ratio of fundamental airgap flux by a sine-wave armature m.m.f. in the q-axis to the maximum value in the direct axis due to the same m.m.f.

C_m = the ratio of the fundamental airgap flux by a sine-wave armature m.m.f. in the d-axis to that by the field of the same m.m.f.

Then by definition,

$$C_{d1} = C_m C_1, \quad \text{or} \quad C_m = C_{d1} / C_1$$

Since the harmonic fluxes and voltages are neglected, the per unit direct-axis armature-reaction reactance X_{ad} is given by

$$X_{ad} = \frac{\text{fundamental reactive voltage produced by rated armature current in direct axis}}{\text{rated voltage}}$$

$$= \frac{\text{fundamental flux due to } A_1 \text{ in direct axis}}{\text{fundamental flux due to } F \text{ in the field}}$$

$$= \frac{C_{d1} A_1}{C_1 F} = \frac{C_m A_1}{F} = \frac{A_{1f}}{F} \quad (3.1)$$

Similarly

$$X_{aq} = \frac{C_{q1} A_1}{C_1 F} = \frac{C_{q1}}{C_{d1}} X_{ad} \quad (3.2)$$

3. 3. GRAPHICAL DETERMINATION OF FLUX-DISTRIBUTION COEFFICIENTS

The flux-distribution coefficients can be obtained by Fourier's Harmonic Analysis of the flux wave, as determined by (a) test, (b) flux plotting, or (c) combining the m.m.f. wave with a permeance curve. Weiseman (22), on a basis of comprehensive flux-plotting, obtained curves for the determination of flux-distribution coefficients for poles of a variety of shapes. Doherty and Nickle (14) derived methods for the determination of permeance curves, and expressed the reactances in terms of permeance-curve coefficients.

The flux distribution depends on the shape of the pole. Four basic pole dimensions affect the flux distribution; viz., the pole pitch, the pole arc, the minimum gap and the maximum gap. See Fig. 3.5.

The shape of the curve of the pole face also affects the flux distribution. But the effect is usually very small, and because the profile of a pole face of given pole arc fraction, minimum gap and maximum gap is defined within comparatively narrow limits, the effect can be neglected and the pole face curve rarely needs to be taken into consideration.

Naturally, only the relative magnitudes of the four dimensions affect the flux distribution: all flux waves for the same relative magnitudes of these four dimensions will have the same shape, and will differ only in scale. Therefore to find the flux distribution of different pole shapes, it is only necessary to plot flux shape for values of the ratios

$\frac{\text{pole arc}}{\text{pole pitch}}$, $\frac{\text{maximum gap}}{\text{minimum gap}}$, and $\frac{\text{minimum gap}}{\text{pole pitch}}$.

Wiesmann (22) constructed flux plots for these ratios for the following conditions:

- (a) field excitation only;
- (b) direct-axis armature reaction only;
- (c) quadrature-axis armature reaction only.

In (b) and (c) sine distribution of the m.m.f. was assumed. From analysis of the flux waves resulting under these conditions for various appropriate values of the geometric ratios, Wiesemann obtained the magnitudes, not only of the fundamental, but also of the third harmonic, expressed as flux distribution coefficients $C_1, C_3; C_{d1}, C_{d3}; C_{q1}$, and C_{q3} . C_1, C_{d1} , and C_{q1} were defined above: C_3, C_{d3} , and C_{q3} are defined as follows:-

C_3 = ratio of third harmonic to fundamental of airgap flux produced by the field;

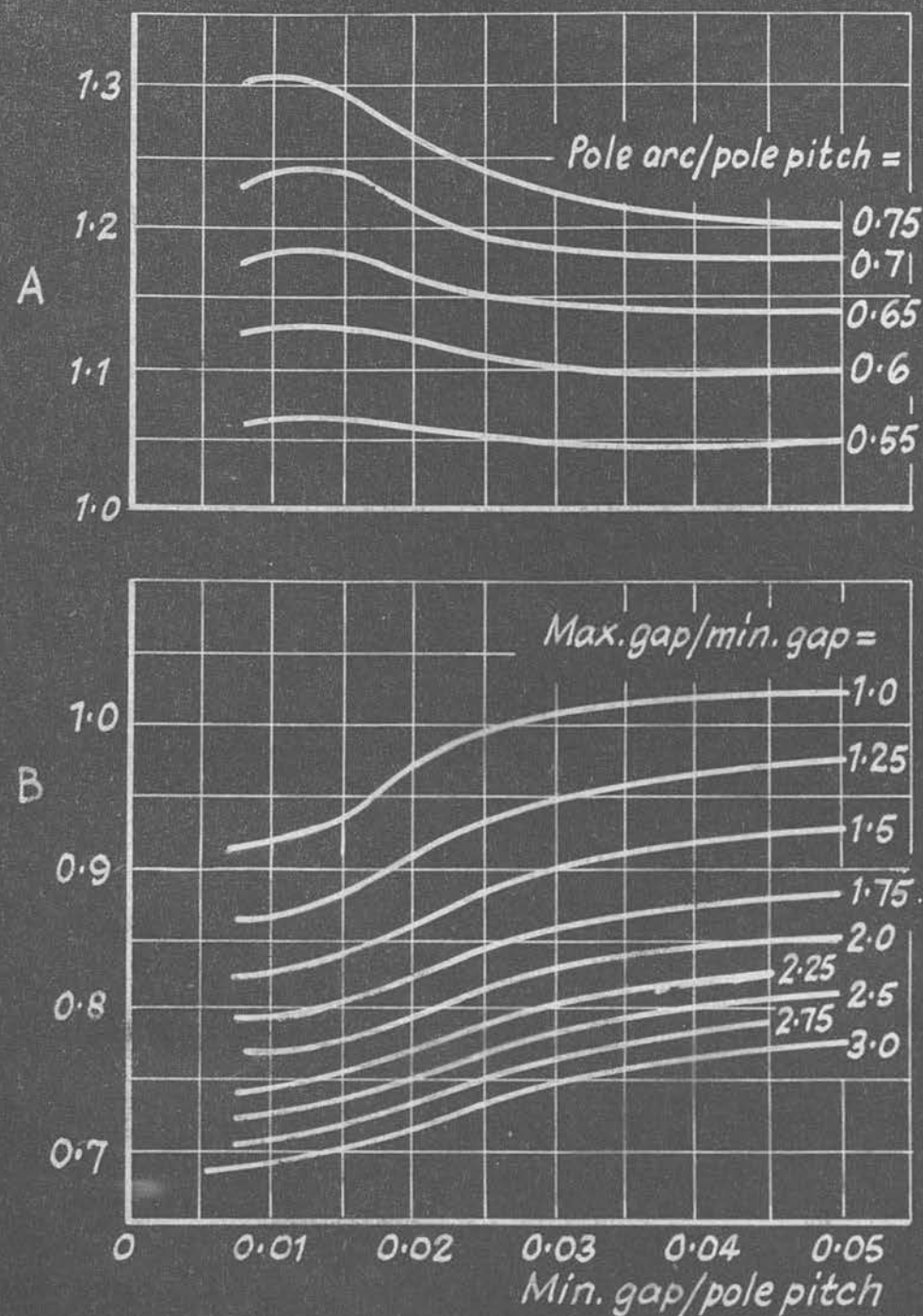


FIG. 3.6. Fundamental of no-load flux wave in gap under conditions of Fig. 3.1. Peak of actual flux wave = 1: then —
Fundamental $C_1 = A \times B$.

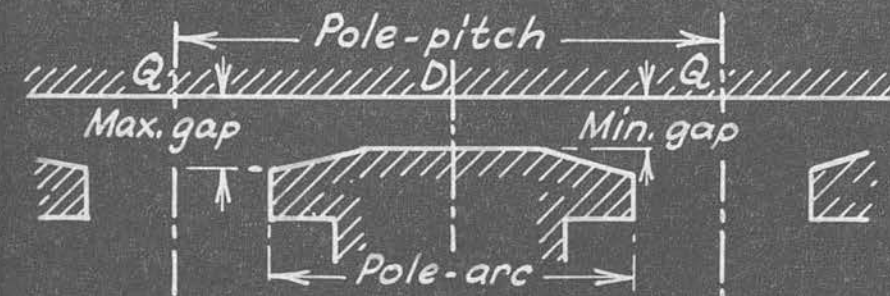


FIG. 3.5. Pole dimensions affecting flux distribution.

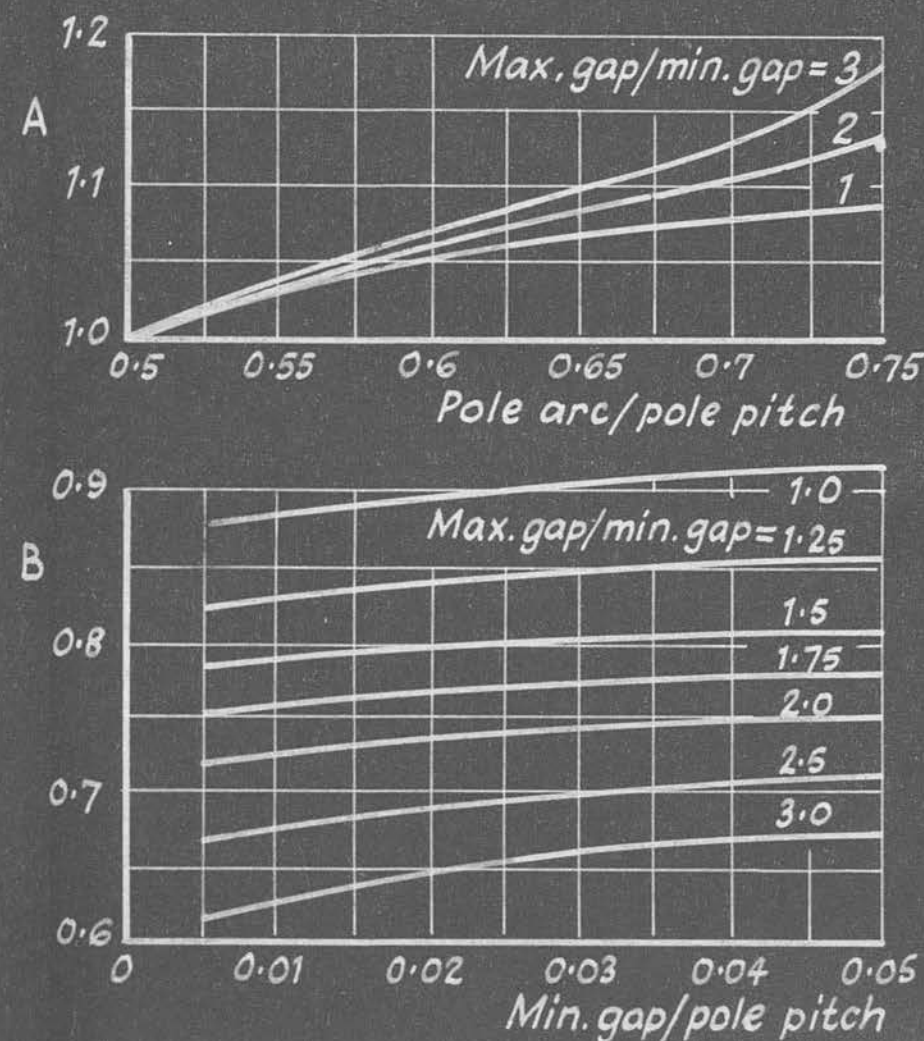


FIG. 3.7. Fundamental of D-axis flux wave in gap under conditions of Fig. 3.2. Peak of actual flux wave = 1: then —
Fundamental $C_{d1} = A \times B$.

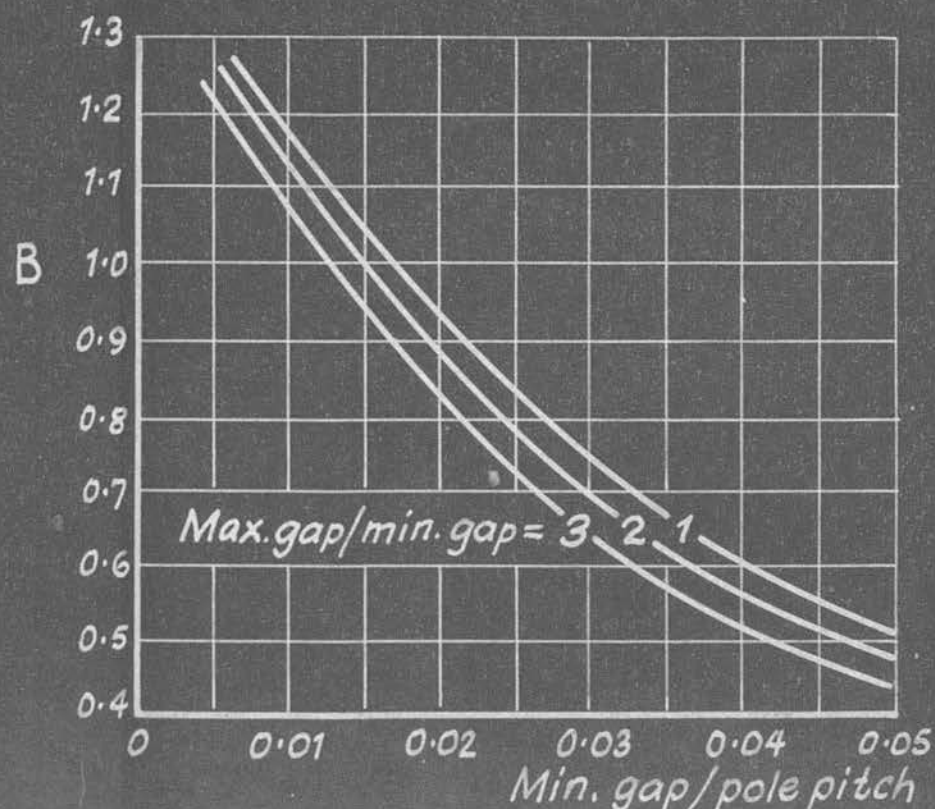
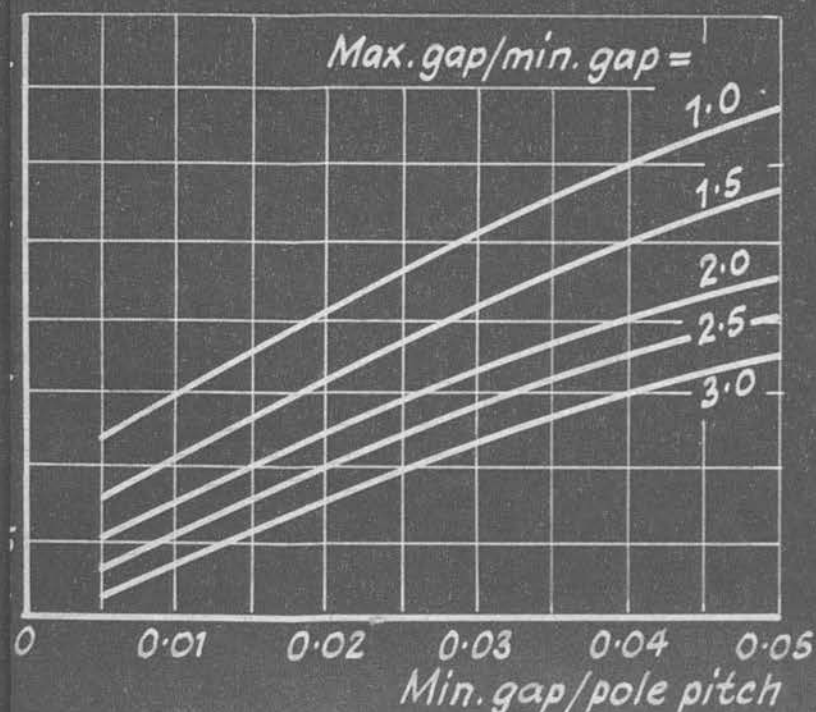
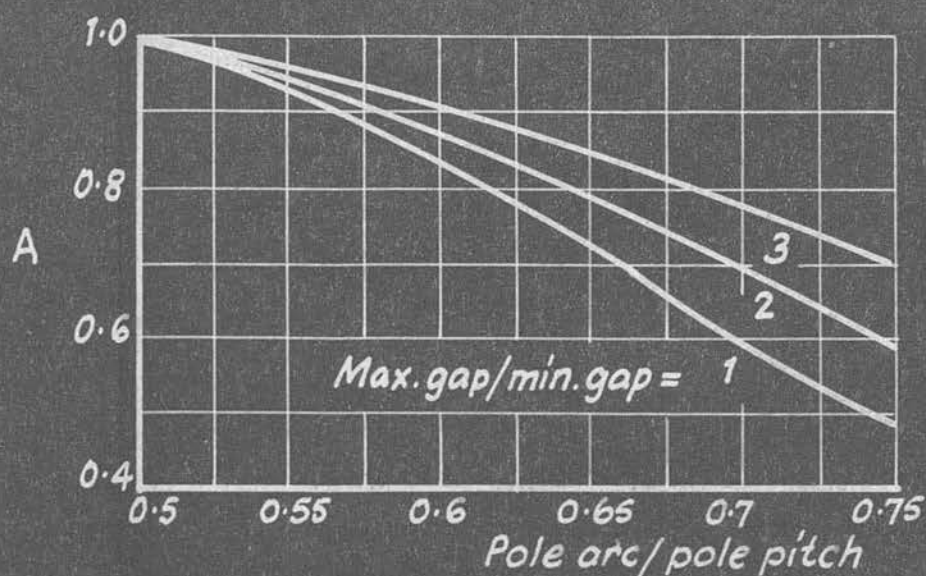
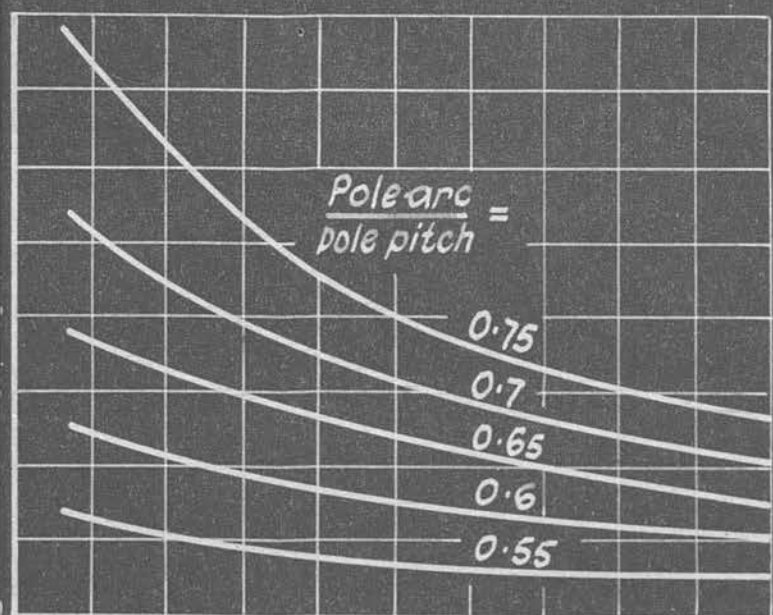


FIG. 3.8. Fundamental of Q-axis flux wave in gap under conditions of Fig. 3.3. Peak of armature m.m.f. = 1; then—
Fundamental $C_{q1} = A \times B$.

FIG. 3.9. Third-harmonic of Q-axis flux wave in gap under conditions of Fig. 3.3. Peak of fundamental flux component = 1; then—
Third harmonic $C_{q3} = A \times B$.

C_{d3} = a ratio of third harmonic to fundamental of airgap flux by a sine-wave armature m.m.f. in the direct axis;

C_{q3} = ratio of third harmonic to fundamental of airgap flux by a sine-wave armature m.m.f. in the quadrature-axis.

For each of the six coefficients Wieseemann plotted a family of curves from which the value of the coefficient can be found for values of the above-mentioned three geometric ratios within the range of ordinary design. The curves for C_1 , C_{d1} , C_{q1} , and C_{q3} are reproduced in the Figs. 3.6-9. Since C_3 and C_{d3} are relatively very small, they usually can be neglected and the curves for them are not reproduced here.

3. 4. DOHERTY AND NICKLE'S ANALYSIS

3. 4. (1) METHOD

Doherty and Nickle (14) derived accurate quantitative expressions for fundamental-frequency voltages produced not only by the space fundamental of armature m.m.f., but also by all its space harmonics. The ideal case with the assumption of a sinusoidal distribution of armature m.m.f. is then only a special case of the more general conditions analysed by them.

In their analysis the armature current i is first split into direct-axis and quadrature-axis components i_d and i_q , which set up fundamental and harmonic sinusoidal space distributions of m.m.f. which rotate at various speeds with respect to the armature, and at still different speeds with respect to the poles. Any sinusoidal wave rotating with respect to the poles can be resolved into two components which are stationary relative to the poles, and are in space- and time-quadrature with each other. In this way all the different harmonic distributions of m.m.f. due to the entire armature current can be resolved, with respect to the pole system, into a series of stationary but alternating m.m.f.'s. To each of them there will correspond some permeance distribution P . The product of the magnitude of a stationary m.m.f. and the corresponding ordinates of its appropriate permeance curve will then give, for every point in the gap, the radial field intensity there. The fluxes so developed will in turn generate in the armature winding voltages covering a considerable frequency range. As pointed out by Doherty and Nickle, the harmonic-frequency voltages are of relatively small magnitude compared with those of fundamental frequency. Consequently the harmonics were considered as negligible, and attention was confined to formulae for fundamental-frequency voltages only.

3. 4. (2) RESULTS

It was found by Doherty and Nickle that the gap permeance curves are not the same for all harmonics of the m.m.f., wave, but

are approximately the same for the sine and cosine distributions (relative to the pole axis) of a given m.m.f. harmonic: i.e.

$$P_s'' \doteq P_c''$$

where P_s is the permeance to sine and P_c the permeance to cosine m.m.f. distributions, and n denotes the order of the harmonic.

Since the airgap is symmetrical about the direct- and quadrature-axes, so also are the permeance curves, which comprise a mean value and even harmonics only. The permeance distribution equations are of the form

$$P'' = P_0'' + P_2'' \cos 2\alpha + P_4'' \cos 4\alpha + \dots$$

where α is the electrical space angle measured from the direct-axis (i.e., the pole centre) where the permeance is a maximum.

It is notable that the general equations derived by Doherty and Nickle indicate that the orders of the space harmonics of permeance which produce fundamental voltages are only the zero and the second, so that only P_0'' and P_2'' enter into the equations of the fundamental voltages. Therefore it is only necessary to analyze the permeance curve for the average and second harmonic in order to find the fundamental voltages.

Let

A = maximum fundamental armature amper-turns

$$= 2.7 n i k_p k_d ;$$

$$A_d = 2.7 n i_d k_p k_d ;$$

$$A_q = 2.7 n i_q k_p k_d ;$$

$$E_0 = \text{r.m.s. rated voltage;}$$

$$B_1 = \text{flux density amplitude of the space fundamental of the flux wave of the field at rated voltage and no load;}$$

$$P_m'' = \text{permeance distribution where } n \text{ denotes the order of m.m.f. harmonic and } m \text{ that of the permeance curve.}$$

Then the reactive fundamental voltage produced by direct-axis component of fundamental armature m.m.f., as derived by Doherty and Nickle, is

$$E_d = -j \frac{0.4\pi A_d E_0}{B_1} (P_0' + \frac{1}{2} P_2') \quad (3.3)$$

The reactive fundamental voltage produced by quadrature-axis component of fundamental armature m.m.f. is

$$E_q = -j \frac{0.4\pi A_q E_0}{B_1} (P_0' - \frac{1}{2} P_2') \quad (3.4)$$

The reactive fundamental voltages produced by harmonics of armature m.m.f. in both axes are listed in Table 3.1,

where k_n is the combined pitch and breadth factor ($= k_p k_d$) of the n th harmonic.

As in a three-phase machine there are no harmonic m.m.f.'s. of order $3n$, no fundamental voltages can be produced by them.

Now as the armature reaction reactance is defined as the ratio of the fundamental reactive voltage produced by a sine-wave m.m.f. to the armature-current responsible for the m.m.f. wave, the reactive voltages produced by the harmonics of the m.m.f. wave are not included in the armature-reaction reactance drop, but are considered as a part of armature leakage reactance drop, or airgap leakage reactance drop which may in turn be divided into belt and zigzag leakage reactance drops.

$$\therefore X_{ad} = C \left(P_0' + \frac{1}{2} P_2' \right) \quad (3.5)$$

and

$$X_{aq} = C \left(P_0' - \frac{1}{2} P_2' \right) \quad (3.6)$$

where C is a constant which can be found from the formulae in Table 3.1.

It is then evident that for an ideal machine of sine-wave m.m.f. there is no fundamental airgap leakage reactance drop and the airgap leakage flux produces only harmonic reactive voltages which may be neglected.

It is to be noted from the Table that while X_{ad} is greater than X_{aq} , the direct-axis airgap leakage reactance is smaller than that of the quadrature-axis. However, as the difference is small and the airgap leakage is only part of the total armature leakage, the main component of which (slot and end-winding leakage), is very nearly constant, the total leakage reactance is usually assumed to be the same for both axes. The nature of leakage reactance will be discussed in greater detail later.

In the case of a cylindrical-rotor machine, the permeance is constant along the gap periphery and the permeance curve contains no harmonics. Therefore $P_2'' = 0$ and all the terms containing P_2'' in Table 3.1 vanish so that the reactances in the direct- and quadrature-axis become equal.

3. 5. PERMEANCE CURVES

In the previous section formulae were given for the fundamental reactive voltages expressed in terms of the coefficients of

TABLE 3.1. MAGNITUDES AND PHASES OF FUNDAMENTAL VOLTAGES DUE TO ARMATURE REACTION

Fundamental voltage due to n^{th} flux harmonic

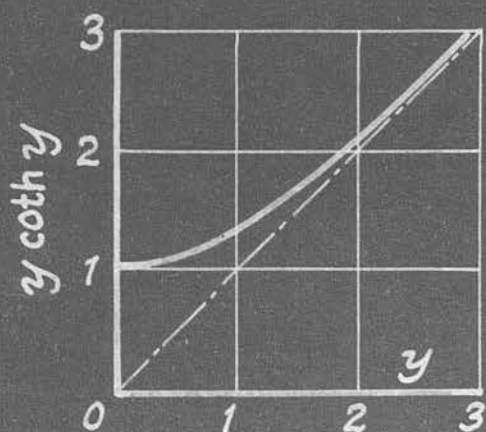
(i) produced by i_d : $-j \frac{0.4\pi A_d e_o}{B_1} \times F_{dn}$

(ii) produced by i_q : $-j \frac{0.4\pi A_q e_o}{B_1} \times F_{qn}$

Order of m.m.f. harmonic	F_{dn}	F_{qn}
1	$(P_0^I + \frac{1}{2} P_2^I)$	$(P_0^I - \frac{1}{2} P_2^I)$
5	$(\frac{P_0^V}{25} k_5^2 - \frac{P_2^V}{70} k_5 k_7) \frac{1}{k_1^2}$	$(\frac{P_0^V}{25} k_5^2 + \frac{P_2^V}{70} k_5 k_7) \frac{1}{k_1^2}$
7	$(\frac{P_0^{VII}}{49} k_7^2 - \frac{P_2^{VII}}{70} k_5 k_7) \frac{1}{k_1^2}$	$(\frac{P_0^{VII}}{49} k_7^2 + \frac{P_2^{VII}}{70} k_5 k_7) \frac{1}{k_1^2}$
11	$(\frac{P_0^{XI}}{121} k_{11}^2 - \frac{P_2^{XI}}{286} k_{11} k_{13}) \frac{1}{k_1^2}$	$(\frac{P_0^{XI}}{121} k_{11}^2 + \frac{P_2^{XI}}{286} k_{11} k_{13}) \frac{1}{k_1^2}$
13	$(\frac{P_0^{XIII}}{169} k_{13}^2 - \frac{P_2^{XIII}}{286} k_{11} k_{13}) \frac{1}{k_1^2}$	$(\frac{P_0^{XIII}}{169} k_{13}^2 + \frac{P_2^{XIII}}{286} k_{11} k_{13}) \frac{1}{k_1^2}$
17	$(\frac{P_0^{XVII}}{289} k_{17}^2 - \frac{P_2^{XVII}}{646} k_{17} k_{19}) \frac{1}{k_1^2}$	$(\frac{P_0^{XVII}}{289} k_{17}^2 + \frac{P_2^{XVII}}{646} k_{17} k_{19}) \frac{1}{k_1^2}$
-	-----	-----

$k_n = k_{pn}$. k_{dn} is the combined winding factor for the n^{th} harm. Harmonic m.m.f.'s of order $3n$ do not occur in a symmetrical 3-phase machine.

FIG. 3.10. Curve of $y \coth y$.



permeance curves. Methods of determining the permeance curves are now discussed.

Since the direction of flux in the airgap is not necessarily radial*, the permeance may not be constant along a given radial line. Therefore the permeance in the airgap is defined as the radial permeance at the armature surface, i.e., as the ratio of radial flux density at the armature surface to the m.m.f., (0.4π times the effective ampere-turns).

The field m.m.f., is assumed to be a square wave of full pole pitch (See Fig. 3.1). This is justified because the permeance of the iron part of the magnetic path is so much greater than that of the airgap that the drop of m.m.f. in the iron part may be neglected and the armature surface assumed to be at zero magnetic potential. Then when the machine is excited only by the field the radial flux density curve at the armature surface is the basic permeance curve.

There are two methods for the determination of the permeance curves.

(A) The first method is by flux plotting or by actual test so that the radial flux density at the armature surface can be determined. Then the flux density curve divided by the m.m.f. curve at a corresponding point will give the permeance for the given m.m.f. wave.

(B) The second method was given by Doherty and Nickle and may be called "equivalent airgap curve method", as an equivalent airgap is to be determined first and then permeance curves for different harmonics obtained therefrom.

It was shown by Doherty and Nickle that the permeance of a uniform gap is

$$P = \frac{n P_1}{2 R_a} \cdot \frac{R_a^{nP_1} + R_f^{nP_1}}{R_a^{nP_1} - R_f^{nP_1}} \quad (3.7)$$

where n = order of harmonic of m.m.f. wave

P_1 = number of poles

R_a = radius of armature

R_f = radius of rotor

If the airgap is relatively very small in comparison with the armature diameter, i.e., $\frac{g}{R_a}$ approaches zero as a limit, the permeance becomes

$$P = \frac{1}{g} \cdot \frac{n\pi g}{\gamma} \coth \frac{n\pi g}{\gamma} = \frac{1}{g} y \coth y \quad (3.8)$$

where g = effective airgap
 γ = pole pitch

* The flux lines must however, enter the boundary ferromagnetic surfaces normally if saturation is negligible.

The value of $y \coth y$ is plotted against y in Fig. 3.10.

$$\begin{aligned} \text{When } y &= 0, \\ y \coth y &= 1, \text{ and} \\ P &= \frac{1}{g} \end{aligned}$$

Therefore for a uniform m.m.f. wave, i.e., zero harmonic, or $n = 0$, the permeance is equal to the reciprocal of the airgap length. For the harmonics of m.m.f., with the airgap small compared with the harmonic pole pitch, the permeance is also approximately given by the reciprocal of the airgap length.

Thus the reciprocal of the permeance curve for zero order harmonic (which can be obtained either by flux plotting or by experiment) gives an equivalent airgap curve. The equivalent airgap may be considered as a transformation of the salient-pole machine into a shaped-rotor machine with non-uniform airgap and with the same permeance curves as the prototype salient-pole machine.

From the equivalent curve and Fig. 3.10 values of g and $y \coth y$ can be obtained for any position at the armature surface and for any harmonic of the m.m.f., wave respectively. Then from eq. (3.8) the permeance curve for any harmonic of m.m.f. curve can be determined.

As pointed out by Doherty and Nickle, if the ratio of the maximum equivalent airgap to the radius is less than say 0.15, the error in the calculated permeance will probably be less than 10%, the calculated value being too low. If the ratio is greater than 0.15, eq. (3.7) should be used instead of eq. (3.8)

C H A P T E R I V

SYNCHRONOUS - MACHINE CONSTANTS

4. 1. GENERAL

In the previous chapters the fundamental principles of the two-reaction theory were given, with an analysis of the magnetic flux distribution in a salient-pole machine, leading to the direct-axis and quadrature-axis synchronous reactances X_d and X_q . While these two constants are sufficient for the study of the steady-state operating conditions of a salient-pole machine, transient conditions require more detailed investigation. For the characteristics and performance of a salient-pole machine in general it is essential to obtain further coefficients in terms of which the machine's transient characteristics and performance are expressible. The definitions of these constants are here discussed.

4. 1. (1) TRANSIENTS

The synchronous reactance is composed of two parts, the armature-reaction reactance and the armature-leakage reactance. The armature-reaction reactance has different values, respectively X_{ad} and X_{aq} , in the direct- and quadrature-axes, the latter normally being smaller than the former because of the lower permeance in the quadrature-axis. The leakage reactance varies only slightly in the two axes: it is usually a little larger in the q-axis than in the d-axis. But as the variation is rather small and the leakage reactance itself is only a small part of the synchronous reactance its variation is small enough to be neglected.

The synchronous reactance is the ratio of the induced fundamental e.m.f. to the fundamental armature current. This refers to steady state only. In steady-state operation the armature flux is stationary with respect to the poles and therefore the flux linkages with the rotor circuits are constant. But under transient conditions there is change of mutual flux between the armature and rotor circuits, and the rotor circuits will act against the change of flux according to Lenz Law. There will be currents induced in the rotor circuits which will oppose the flux changes of the armature current. Then the apparent flux produced by unit armature current is smaller under transient than under steady-state conditions, and so also are the flux linkage and the reaction voltage. Therefore the transient reactances are smaller than steady-state, or synchronous reactances. The reactances under transient conditions when only the effect of the field-winding circuit is considered are usually termed transient, while when the effect of both the field-winding circuit and additional rotor circuits (such as damper bars) are considered, they are termed

sub-transient. Thus according to the method of application in time of armature current, the reactances may be classified as sustained, transient and sub-transient.

4. 1. (2) UNBALANCE

For unbalanced conditions of operation the armature currents may be resolved by the method of symmetrical components into positive-, negative- and zero-phase-sequence currents. Under balanced conditions of operation only the positive-phase-sequence currents exist and the synchronous reactances are therefore based thereon. For negative- and zero-phase-sequence currents there are corresponding reactance values.

If the rotor is kept stationary and a ~~line~~-to-neutral or ~~line~~-to-~~line~~ current is applied to the armature, the operative reactances are termed static reactances. The relations between static, sub-transient and zero-phase-sequence reactances will be discussed later: The sub-transient and zero-phase-sequence reactances can be determined from static reactances by test.

4. 1. (3) TIME-CONSTANTS

The transient currents in the armature and rotor circuits will decay from their initial values to steady-state values. They may each be resolved into a steady-state component and several transient components which decay according to appropriate time-constants. The time-constant of the armature circuit is uni-valued, but the rotor circuits have both open-circuit and short-circuit time constants according to whether the armature is open- or short-circuited, and also have different values in the direct and quadrature-axes. The direct component and even harmonics in the armature current decay according to the armature time constant, the odd harmonics according to the rotor time-constants. In the rotor circuits the direct component and the even harmonics decay according to their own time-constants, and the odd harmonics according to the armature time-constant.

4. 2. SYNCHRONOUS, ARMATURE-REACTION AND LEAKAGE REACTANCES

4. 2. (1) DEFINITIONS

The direct-axis synchronous reactance X_d is the ratio of the fundamental reactive armature voltage produced by a sustained direct-axis component of armature current applied, to the value of that component of current at rated frequency and with the rotor running at synchronous speed.

The quadrature-axis synchronous reactance X_q is defined in an analogous manner.

The armature-reaction reactances X_{ad} and X_{aq} are reactances due to the space fundamental sine wave of airgap flux produced

by the armature current acting alone.

The armature leakage reactance X_l is the difference between the total, or synchronous, armature reactance and the reactance of armature reaction.

4. 2. (2) LEAKAGE REACTANCE

This may be separated into the following four parts, associated with

- (a) slot leakage;
- (b) end-winding leakage;
- (c) zig-zag leakage; and
- (d) belt leakage.

The belt and zig-zag reactances together are called the differential leakage reactance or airgap leakage reactance.

The slot leakage reactance is that due to the flux crossing the slots below the armature surface.

The end-leakage reactance corresponds to the flux linking the end-windings alone.

The differential leakage reactance is the total reactance due to harmonic fluxes crossing the airgap that are not of the fundamental number of poles.

The zig-zag leakage reactance is that due to the space harmonics of the airgap flux, which induces fundamental frequency voltages in the armature. This includes the flux which crosses from tooth to tooth in the airgap and interpolar spaces without actually reaching the field surface, and so takes up the leakage flux at the point at which the slot leakage left it.

The belt leakage reactance is the reactance due to all the remaining fundamental frequency voltage producing space harmonics of the airgap flux caused by armature current. It is the additional reactance that an actual winding has above that which it would have if there were as many phases as slots per pole.

In fact, no rigid dividing line can be drawn between the zig-zag and belt leakages, as they are mutually dependent. The reason for making a distinction is that, for ordinary design proportions, a change in the number of slots affects only the zig-zag leakage, while a change in the winding pitch affects only the belt leakage, so that the independent effects of number of slots and of pitch can be more easily dealt with when the two parts are considered separately.

The leakage reactance is usually assumed to be constant, though in reality the differential leakage is bigger in the quadrature-axis than in the direct-axis. The slot leakage and the

end leakage can be taken as constant, because the paths of the flux are substantially independent of the armature position with respect to the poles, except for the secondary influence of the tooth saturation, and therefore the permeances are nearly constant.

As to the differential leakage, the direct-axis component of the current is maximum when the slots are midway between the poles, where the permeance is a minimum, and the quadrature-axis component of the current is maximum when the slots are directly under the pole centre, where the permeance is a maximum. Therefore the differential leakage is larger in the quadrature- than in the direct-axis.

Karapetoff (15) showed that the leakage reactance is of a variable nature depending upon the angle of lag of the armature current on the nominal voltage. He assumed that the leakage inductance, which is variable with the rotor position, consists of a constant and a second harmonic. In reality the leakage reactance varies with the permeance which also contains even harmonics of higher order than the second. But, as shown by Doherty and Nickle, the only harmonics in the permeance curve that affect the fundamental induced voltage are the zero- and second-order terms: the assumption made by Karapetoff is consequently justified.

It was remarked in Chapter III that Doherty and Nickle derived formulae for the fundamental voltages induced by the air-gap flux produced by the fundamental and space harmonics of the armature m.m.f. The space fundamental of airgap flux is produced by the fundamental of armature m.m.f., while the space harmonics of **armature** m.m.f. produce only space harmonics of airgap flux. (The fundamental of armature m.m.f. produces space harmonics of airgap flux, too. But these rotate at synchronous speed with respect to the armature and produce only harmonic frequency voltages, which are to be neglected). Since by definition armature-reaction reactance is that part due to the space fundamental of airgap flux, it is the fundamental voltage per unit current induced by the airgap flux produced by the space fundamental of armature m.m.f. Therefore all the fundamental-reactive-voltage-inducing airgap fluxes produced by the space harmonics of armature m.m.f. are airgap leakage flux. In the case of an ideal machine with a sine-wave armature m.m.f. there is no airgap leakage flux producing fundamental voltages.

The direct-axis airgap leakage reactance given by Doherty and Nickle is

$$\chi_{ltd} = \frac{P_0^v C_5^2}{25} - \frac{P_2^v C_5 C_7}{70} + \frac{P_0^{III} C_7^2}{49} - \frac{P_2^{III} C_5 C_7}{70}$$

+

(4.1)

The quadrature-axis airgap leakage reactance is

$$X_{ltq} = \frac{P_0^v C_5^2}{25} + \frac{P_2^v C_5 C_7}{70} + \frac{P_0^m C_7^2}{49} + \frac{P_2^m C_5 C_7}{70} + \dots \quad (4.2)$$

The negative signs of half of the terms in (4.1) show that X_{ltq} is smaller than X_{ltq} . But as pointed out by Doherty and Nickle, the maximum variation in leakage reactance with pole position is of the order of $\pm 1\%$ of the armature reaction reactance; and in many cases, on account of fractional pitch, etc., it would be practically zero. Therefore the variation of leakage reactance may be neglected and it is fully justified to assume the leakage reactance to be constant.

4. 3. TRANSIENT REACTANCES

In steady-state operation the mutual flux between armature and rotor circuits moves at synchronous speed and is therefore stationary with respect to the rotor. So there is no change of linkage in the rotor circuits and the synchronous reactance is equal to the sum of armature-reaction reactance and armature leakage reactance. But under transient conditions, such as short-circuit, sudden change of power angle, starting, or a current or voltage suddenly applied to the armature, the change of armature current tends to change the linkage with the rotor circuits. According to the Lenz Law, c.m.f.'s., are induced in the rotor circuits, which cause currents to flow the effect of which is initially to neutralize the effect of the armature current so that the flux linkage in each rotor circuit does not alter at the instant the transient occurs.

4. 3. (1) NO ADDITIONAL ROTOR CIRCUITS

Consider the simplest case, in which there are no rotor circuits additional to the normal field winding. A transient field current is induced to neutralize the change of flux linkage produced by the transient armature current. If there were no field leakage reactance, and the flux distribution due to the field current alone were the same as that due to the armature current alone, the mutual flux produced by the armature current would be entirely neutralized by the flux produced by the field current, and the armature current would attain such a value that the armature leakage reactance voltage drop would be equal to the applied voltage. But in reality the flux distribution due to the field current is never identical with that due to the armature current

and there is always some field leakage flux, so that the armature current does not attain so high a value. The armature current and field current will be of such values that the net linkage in the armature circuit will balance the applied voltage and the net linkage in the field circuit will remain the same as at the instant just before the transient occurs.

Because of this effect of induced field current the apparent total reactance in the armature circuit, i.e., the reactive voltage per unit armature current, will be smaller than the synchronous reactance. The reduced value is termed the transient reactance, its magnitude being somewhere between the limits of armature leakage reactance and synchronous reactance.

The transient reactance of a machine in the direct-axis, X_d' (or in the quadrature-axis, X_q') is therefore defined as the ratio of positive phase-sequence reactive voltage due to the sudden application of a positive phase-sequence current in the direct (or quadrature) axis, to that current, if there are no additional rotor circuits.

Under transient conditions the armature and field resemble the primary and secondary of a short-circuited transformer. Therefore if the field circuit is changed into armature terms, the armature and field together can be represented by an equivalent circuit of the transformer type.

If the turn-ratio is so chosen that the mutual reactance is equal to the armature reaction reactance, the equivalent circuit for armature and field without additional rotor circuits will be as shown in Fig. 4.1,

where r_a = armature resistance;
 X_l = armature leakage reactance;
 R_{fd} = field resistance;
 X_{fd} = field total reactance;
 X_{ad} = direct-axis armature reaction reactance;
 X_{afd} = mutual reactance between armature and field.

From Fig. 4.1, it is apparent that the direct-axis synchronous reactance X_d is equal to $X_l + X_{ad}$.

Neglecting the field resistance, which is usually small compared with the mutual reactance, the direct-axis transient reactance can be found from the circuit as

$$\begin{aligned} X_d' &= X_l + \frac{1}{\frac{1}{X_{afd}} + \frac{1}{X_{fd} - X_{afd}}} = X_l + \frac{X_{afd}(X_{fd} - X_{afd})}{X_{fd}} \\ &= \frac{(X_l + X_{ad})X_{fd} - X_{afd}^2}{X_{fd}} = X_d - \frac{X_{afd}^2}{X_{fd}} \end{aligned} \quad (4.3)$$

Since there is no field winding in the quadrature-axis, the quadrature-axis transient reactance is equal to the quadrature-axis synchronous reactance.

4. 3. (2) WITH ADDITIONAL ROTOR CIRCUITS

In most synchronous machines the rotor possesses well- or ill-defined additional circuits such as damper bars, field collars, solid pole-shoes, etc. Under transient conditions these will carry induced currents, of such magnitude that their linkages will remain constant at the instant the transient conditions are imposed. Equivalent circuits for an armature with field and additional rotor circuits were developed by Linville, Liwschitz, and Rankin. Rankin's equivalent circuits are the most complete and useful, and will be discussed in Chapter VI.

The resistance of additional rotor circuits is usually much bigger than that of the field circuit when all are expressed in armature terms. As the time-constant of a transient current varies inversely as the resistance, the time constants of the additional rotor circuits are usually much shorter than those of the field circuit. The transient currents in the additional rotor circuits thus decay very rapidly, usually in a few cycles, while the transient current in the main field may persist for a few seconds.

In order to distinguish between the effects of field circuit and of additional rotor circuits, the effect of field and additional rotor circuits together is termed sub-transient and that due to the field alone is termed transient. The sub-transient reactance of a machine in either the direct or the quadrature-axis is defined as the ratio of positive-phase-sequence reactive voltage due to the sudden application of a positive phase-sequence current to that current, while transient reactance is the corresponding apparent value that obtains after the "short-time-constant" transients have decayed to negligible magnitudes.

4. 4. NEGATIVE PHASE-SEQUENCE REACTANCE

The negative-phase-sequence reactance of a machine is the ratio of the negative phase-sequence fundamental reactive armature voltage produced by a negative phase-sequence fundamental armature current applied, to that current, when the machine is running at synchronous speed.

When a negative-phase-sequence fundamental armature current flows, a constant m.m.f. wave is formed which rotates at synchronous speed with respect to the armature in the negative direction and therefore at double synchronous speed with respect to the rotor. This rotating m.m.f. wave may be resolved into

two double-frequency pulsating waves in space and time quadrature in the direct and quadrature axes. The pulsating m.m.f. waves combined with the permeance curves give two pulsating flux waves, the fundamentals of which are proportional (or, in per-unit terms, equal) to iX_d'' and iX_q'' . Here the subtransient reactances X_d'' and X_q'' are used instead of the synchronous reactances because the rotating m.m.f. wave is not stationary with respect to the rotor and there are persistent changes of flux linkage in the rotor circuits so that the conditions are such as to conform most closely with those on which the subtransient reactances are based. Now neglecting the harmonics in the flux wave, we can resolve each of the fundamental pulsating flux waves iX_d'' and iX_q'' into two rotating flux waves of half their magnitudes and rotating in opposite directions at double synchronous speed with respect to the rotor. The resultant backward flux wave is of the magnitude

$$\frac{iX_d'' + iX_q''}{2} = i \left(\frac{X_d'' + X_q''}{2} \right)$$

and is at negative synchronous speed with respect to the armature. It generates the negative-phase-sequence fundamental voltage. The negative-phase-sequence reactance is therefore

$$X_2 = \frac{i \left(\frac{X_d'' + X_q''}{2} \right)}{i} = \frac{X_d'' + X_q''}{2} \quad (4.4)$$

The resultant forward flux wave is of the magnitude

$$\frac{iX_d'' - iX_q''}{2} = i \left(\frac{X_d'' - X_q''}{2} \right)$$

at 3 times synchronous speed with respect to the armature and therefore generates a positive-phase-sequence 3rd-harmonic voltage.

The m.m.fs, fluxes, and voltages can be represented by space and time vectors and are shown in the Figs. 4.2-7.

It can be similarly shown that if a negative-phase-sequence fundamental voltage is applied, there will be produced a negative-phase-sequence fundamental current as well as a third-harmonic current. In this case the ratio of the fundamental voltage to the fundamental current is no longer $(X_d'' + X_q'')/2$, but $2X_d''X_q''/(X_d'' + X_q'')$. By definition we only take

$$X_2 = \frac{X_d'' + X_q''}{2}$$

If a negative-phase-sequence fundamental voltage is applied to a machine with an external reactance X_e , the total negative-phase-sequence reactance will become

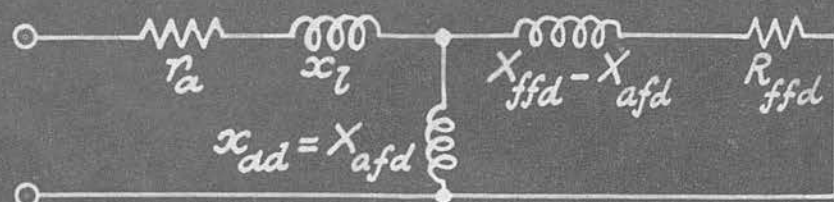


FIG. 4.1. Elementary equivalent circuit of armature and field.

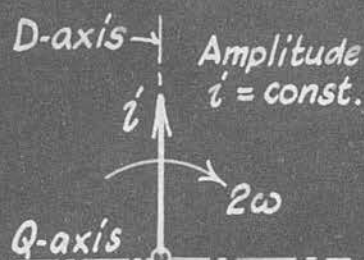


FIG. 4.2. Armature m.m.f. rotating at double syn. speed in negative direction with respect to rotor.

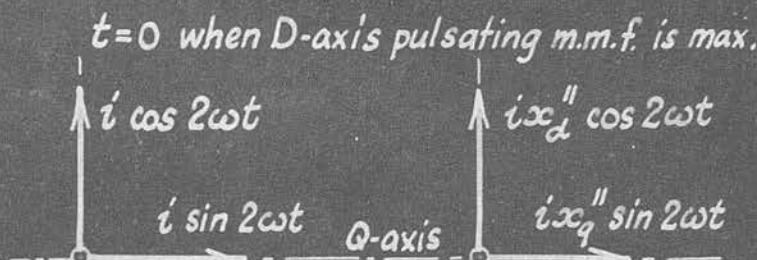


FIG. 4.3. Rotating m.m.f. resolved into two pulsating waves in time and space-quadrature.

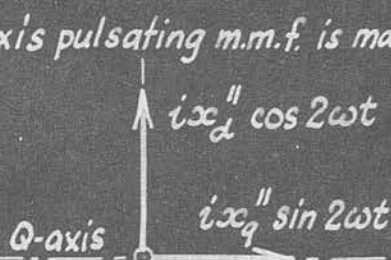


FIG. 4.4. Pulsating m.m.f. waves combined with permeance curve to form pulsating flux waves.

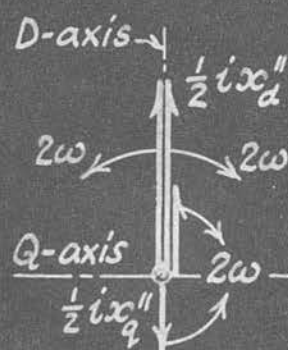


FIG. 4.5. Pulsating flux waves resolved into rotating flux waves of constant magnitude.

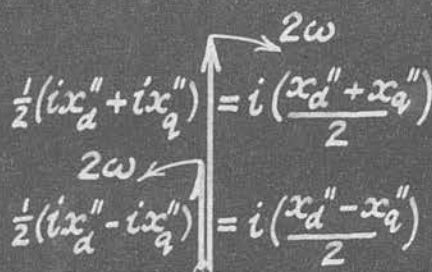


FIG. 4.6. Resultant rotating flux waves with respect to poles.

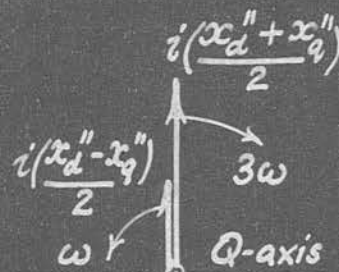


FIG. 4.7. Resultant rotating flux waves with respect to armature.

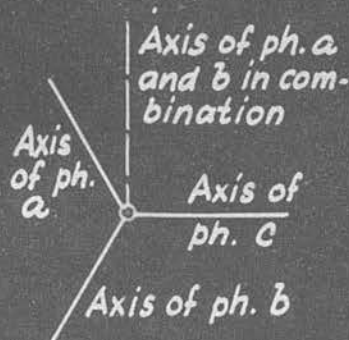


FIG. 4.8. Axis of two phases combined.

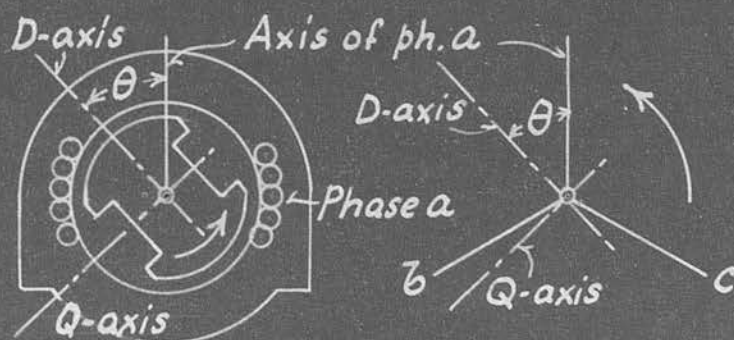


FIG. 4.9. Axes of elementary machine.

$$\begin{aligned} X &= \frac{2(X_d'' + X_e)(X_q'' + X_e)}{X_d'' + X_q'' + 2X_e} \\ &= \frac{2X_d''X_q'' + 2X_e(X_d'' + X_q'') + 2X_e^2}{X_d'' + X_q'' + 2X_e} \end{aligned}$$

If X_e is very large as compared with X_d'' and X_q'' , it approximately becomes

$$\begin{aligned} X &\doteq \frac{2X_e(X_d'' + X_q'') + 2X_e^2}{X_d'' + X_q'' + 2X_e} \\ &= \frac{X_e(X_d'' + X_q'')}{X_d'' + X_q'' + 2X_e} + \frac{X_e(X_d'' + X_q'' + 2X_e)}{X_d'' + X_q'' + 2X_e} \\ &\doteq \frac{X_e(X_d'' + X_q'')}{2X_e} + X_e \\ &= \frac{X_d'' + X_q''}{2} + X_e \end{aligned}$$

Then the reactance of the machine alone becomes

$$X - X_e = \frac{X_d'' + X_q''}{2}$$

which is the same as the negative-phase-sequence reactance X_2 . This is the necessary result because when a fundamental voltage is applied to a machine with a large external reactance, the current approaches a sine-wave and the voltages across both the machine and the external reactance contain a third harmonic of opposite direction and same magnitude so that they neutralize each other.

As a line-to-line short-circuit current contains a negative-phase-sequence component, the A.I.E.E. proposed a method (55) of determining X_2 from a line-to-line short circuit test. It was shown by W.A. Thomas (44) that the negative-phase-sequence reactance of a line-to-line short circuit, i.e., the ratio of the fundamental voltage to fundamental current, is $\sqrt{X_d''X_q''}$, but the negative-phase-sequence reactance determined by this method, i.e., the ratio of measured voltage and current, which are the r.m.s. values and therefore contain the harmonics as well as the fundamental, is equal to $(X_d'' + X_q'')/2$. Therefore this method does not determine the value of the negative-phase-sequence reactance for the operating conditions used, but determines the X_2 as defined above. If the fundamental components of the quadratures measured by means of the A.I.E.E. method are used, a correct value of negative-phase-sequence reactance, namely $\sqrt{X_d''X_q''}$ is determined for that operating condition.

4. 5. ZERO-PHASE-SEQUENCE REACTANCE

The zero-phase-sequence reactance of a machine is the ratio of fundamental phase voltage produced by a zero-phase-sequence

current applied, to that current.

In a three-phase machine the resultant of the fundamental m.m.f. wave produced by a zero-phase-sequence current is zero. So are also the harmonics except the 3rd and its multiples. The 3rd harmonic and its multiples are co-phasal in all three phases. They sum to a pulsating stationary m.m.f. wave with a fundamental pitch equal to one third of the pole-pitch and pulsating at the frequency of the applied current. As this pulsating m.m.f. wave is much smaller than the rotating wave produced by a positive-phase-sequence current, χ_o is usually of a very small magnitude as compared with the synchronous reactance.

Since the pulsating m.m.f. wave causes change of flux linkage in the rotor circuits no matter whether the rotor is stationary or rotating, the rotor circuits always have transient and sub-transient effects on the armature. Therefore χ_o is little affected by whether the rotor is running or at stand still.

4. 6. STATIC REACTANCES

The line-to neutral static reactance is the ratio of the fundamental reactive phase voltage produced by a line-to-neutral fundamental current applied, to that current, when the rotor is kept stationary with respect to the armature.

The direct-axis line-to-neutral static reactance χ_{sd} is that reactance when the direct-axis coincides with the axis of the relevant phase; the quadrature-axis line-to-neutral static reactance χ_{sq} is that reactance when the quadrature-axis coincides with the axis of the phase concerned.

The line-to-line static reactance is the ratio of the line-to-line fundamental reactive voltage produced by a line-to-line fundamental current applied to that current when the rotor is kept stationary with respect to the armature.

The direct-axis line-to-line static reactance χ_{sd} is that reactance when the direct-axis coincides with the axis of the two phases combined. The quadrature-axis line-to-line static reactance χ_{sq} is that reactance when the quadrature-axis coincides with the axis of the two phases combined.

It is to be noted that since in a line-to-line connection one phase is connected to another phase inverted, the axis of the two phases combined is not midway between the axes of the two phases, but is midway between the axis of one phase and the opposite direction of the axis of the other phase, Fig. 4.8.

4. 7. PARK'S FLUX-LINKAGE FORMULAE FOR AN IDEAL SYNCHRONOUS MACHINE

A major contribution to the basic theory of operation of synchronous machines was made by Park (26) in a discussion of the flux-linkages in the phase windings of a machine. From his work the relations between negative- and zero-phase-sequence, static, and sub-transient reactances can be derived. A digest of Park's analysis is given below.

Consider Fig. 4.9, showing an elementary 2-pole machine. Let the d-axis be at the angle θ to the axis of phase A. Then if the rotor is stationary with respect to the armature, θ is a constant. If, however, the rotor is turning at a given speed with reference to the armature, then

$$\theta = \theta_0 + t$$

where t is time expressed in per-unit terms*, or angle in radians.

In general the rotor will carry currents producing in combination an m.m.f. resolvable along the direct and quadrature axes. Let I_d and I_q be the equivalent d- and q-axis rotor currents respectively. Consider the flux-linkages in per unit terms in phase A. There will be

- (i) a contribution from the direct-axis field component which, by virtue of the displacement angle θ , will be proportional to $I_d \cos \theta$. The proportionality factor for flux is the direct-axis armature-reaction reactance X_{ad} , so that the linkage is $X_{ad} I_d \cos \theta$;
- (ii) a similar contribution - $X_{aq} I_q \sin \theta$ from the q-axis field equivalent current;
- (iii) linkage produced by any asymmetry of the phase currents i_a, i_b, i_c , whose vector sum ($i_a + i_b + i_c$) will not then be zero. The linkage is a function of the zero-phase-sequence reactance coefficient X_0 , and becomes $-\frac{1}{3} X_0 (i_a + i_b + i_c)$.
- (iv) contributions by the forward- and backward-moving negative-phase-sequence currents (see (4.4.)).

For phase A these linkages sum to

$$\begin{aligned} \psi_a = & X_{ad} I_d \cos \theta - X_{aq} I_q \sin \theta \\ & - \frac{1}{3} X_0 (i_a + i_b + i_c) - \frac{1}{3} (X_d + X_q) \left[i_a - \frac{1}{2} (i_b + i_c) \right] \\ & - \frac{1}{3} (X_d - X_q) \left[i_a \cos 2\theta + i_b \cos(2\theta - \frac{2}{3}\pi) + i_c \cos(2\theta + \frac{2}{3}\pi) \right] \end{aligned} \quad (4.5)$$

* See Chap V. For constant speed of rotation unit time is that for rotation through one electrical radian, and per-unit time is the number of electrical radians of rotation.

Having regard to the change of phase axis by $2\pi/3$, the linkages in phases B and C are

$$\begin{aligned}\psi_b = & \chi_{ad} I_d \cos(\theta - \frac{2}{3}\pi) - \chi_{aq} I_q \sin(\theta - \frac{2}{3}\pi) \\ & - \frac{1}{3} \chi_0 (i_a + i_b + i_c) - \frac{1}{3} (\chi_d + \chi_q) [i_b - \frac{1}{2}(i_c + i_a)] \\ & - \frac{1}{3} (\chi_d - \chi_q) [i_b \cos(2\theta + \frac{2}{3}\pi) + i_c \cos 2\theta + i_a \cos(2\theta - \frac{2}{3}\pi)] \quad (4.6)\end{aligned}$$

$$\begin{aligned}\text{and } \psi_c = & \chi_{ad} I_d \cos(\theta + \frac{2}{3}\pi) - \chi_{aq} I_q \sin(\theta + \frac{2}{3}\pi) \\ & - \frac{1}{3} \chi_0 (i_a + i_b + i_c) - \frac{1}{3} (\chi_d + \chi_q) [i_c - \frac{1}{2}(i_a + i_b)] \\ & - \frac{1}{3} (\chi_d - \chi_q) [i_c \cos(2\theta - \frac{2}{3}\pi) + i_a \cos(2\theta + \frac{2}{3}\pi) + i_b \cos 2\theta] \quad (4.7)\end{aligned}$$

The d- and q-axis equivalent rotor currents are

$$I_d = \frac{\chi_{afd} I_{fd} + \chi_{ald} I_{ld} + \chi_{azd} I_{zd} + \dots}{\chi_{ad}}$$

$$\text{and } I_q = \frac{\chi_{aiq} I_{iq} + \chi_{azq} I_{zq} + \dots}{\chi_{aq}}$$

where χ_{afd} = mutual reactance between the circuits of the armature and the field winding.

χ_{ald} , χ_{azd} = mutual reactances between the armature and the additional rotor circuits in the D-axis.

χ_{aiq} , χ_{azq} = do. in the Q-axis.

Under steady-state conditions there are no currents in the additional rotor circuits, so that

$$I_d = \frac{\chi_{afd} I_{fd}}{\chi_{ad}} = I_{fd} = \text{constant},$$

$$\text{and } I_q = 0.$$

But under transient conditions there will be induced currents in the additional rotor circuits, and also in the field winding itself. However these currents are taken into account if the synchronous reactances χ_d and χ_q are replaced by the subtransients χ_d'' and χ_q'' , in eqs. (4.5) - (4.7).

4. 8. DERIVATION OF N.P.S. REACTANCE FROM FLUX-LINKAGE FORMULAE

If a negative-phase-sequence fundamental current is applied, the phase currents will be

$$\begin{aligned}i_a &= i \cos t, \\ i_b &= i \cos(t + \frac{2\pi}{3}), \\ \text{and } i_c &= i \cos(t - \frac{2\pi}{3}).\end{aligned}$$

Since there is change of linkages in the rotor circuits due to negative-phase-sequence currents, the subtransient reactances should be used.

Then the linkage in phase A due to the negative-phase-sequence currents is

$$\begin{aligned}
 \psi_a &= -\frac{\chi_o}{3} \left[i \cos t + i \cos \left(t + \frac{2\pi}{3} \right) + i \cos \left(t - \frac{2\pi}{3} \right) \right] \\
 &\quad - \frac{\chi_d'' + \chi_q''}{3} \left[i \cos t - \frac{i \cos \left(t + \frac{2\pi}{3} \right) + i \cos \left(t - \frac{2\pi}{3} \right)}{2} \right] \\
 &\quad - \frac{\chi_d'' - \chi_q''}{3} \left[i \cos t \cos(2t + 2\theta_o) \right. \\
 &\quad \left. + i \cos \left(t + \frac{2\pi}{3} \right) \cos \left(2t + 2\theta_o - \frac{2\pi}{3} \right) + i \cos \left(t - \frac{2\pi}{3} \right) \cos \left(2t + 2\theta_o + \frac{2\pi}{3} \right) \right] \\
 &= -\frac{\chi_d'' + \chi_q''}{3} \cdot \frac{3}{2} i \cos t \\
 &\quad - \frac{\chi_d'' - \chi_q''}{3} \cdot \frac{i}{2} \left[\cos(3t + 2\theta_o) + \cos(t + 2\theta_o) + \cos(3t + 2\theta_o) \right. \\
 &\quad \left. + \cos \left(t + 2\theta_o + \frac{2\pi}{3} \right) + \cos(3t + 2\theta_o) + \cos \left(t + 2\theta_o - \frac{2\pi}{3} \right) \right] \\
 &= -\frac{\chi_d'' + \chi_q''}{2} \cdot i \cos t \\
 &\quad - \frac{\chi_d'' - \chi_q''}{2} \cdot i \cos(3t + 2\theta_o)
 \end{aligned}$$

It is clear from the above equation that the voltage induced by a negative-phase-sequence current consists of a fundamental and a third harmonic and that the negative-phase-sequence reactance χ_2 is equal to $(\chi_d'' + \chi_q'')/2$.

The linkages in phases B and C can be similarly found as

$$\begin{aligned}
 \psi_b &= -\frac{\chi_d'' + \chi_q''}{2} \cdot i \cos \left(t + \frac{2\pi}{3} \right) \\
 &\quad - \frac{\chi_d'' - \chi_q''}{2} \cdot i \cos \left(3t + 2\theta_o - \frac{2\pi}{3} \right) \\
 \psi_c &= -\frac{\chi_d'' + \chi_q''}{2} \cdot i \cos \left(t - \frac{2\pi}{3} \right) \\
 &\quad - \frac{\chi_d'' - \chi_q''}{2} \cdot i \cos \left(3t + 2\theta_o + \frac{2\pi}{3} \right)
 \end{aligned}$$

From the equations of ψ_a , ψ_b , and ψ_c it is also seen that while the fundamental voltage is of negative-phase-sequence as is the current, the third harmonic voltage is of positive sequence.

4. 9. DERIVATION OF STATIC REACTANCES FROM FLUX LINKAGES FORMULAE

If a line-to-neutral fundamental current is applied to phase A

$$i_a = i \cos t, \quad i_b = i_c = 0.$$

Since the rotor is kept stationary with respect to the armature,

$$\theta = \theta_0.$$

And since the flux is pulsating, there is change of linkages in the rotor circuits. Therefore X_d'' and X_q'' should be used.

The flux linkage in phase A due to the line-to-neutral current is therefore, from eq. 4.5.,

$$\begin{aligned} \psi_a = & -\frac{X_0}{3} i \cos t \\ & -\frac{X_d'' + X_q''}{3} i \cos t \\ & -\frac{X_d'' - X_q''}{3} i \cos t \cos 2\theta_0. \end{aligned}$$

From Fig. 4.9, when the direct axis coincides with the axis of phase A, $\theta_0 = 0$.

$$\begin{aligned} \therefore \psi_a = & \left[-\frac{X_0}{3} - \frac{X_d'' + X_q''}{3} - \frac{X_d'' - X_q''}{3} \right] i \cos t \\ = & -\frac{X_0 + 2X_d''}{3} i \cos t \\ \therefore X_{sd} = & \frac{X_0 + 2X_d''}{3} \end{aligned} \quad (4.8)$$

When the quadrature axis coincides with the axis of phase A, $\theta_0 = -\frac{\pi}{2}$.

$$\begin{aligned} \therefore \psi_a = & \left[-\frac{X_0}{3} - \frac{X_d'' + X_q''}{3} + \frac{X_d'' - X_q''}{3} \right] i \cos t = -\frac{X_0 + 2X_q''}{3} i \cos t \\ \therefore X_{sq} = & \frac{X_0 + 2X_q''}{3} \end{aligned} \quad (4.9)$$

If a line-to-line fundamental current is applied to phases A and B,

$$i_a = -i_b = i \cos t, \quad i_c = 0,$$

$$\theta = \theta_0.$$

The flux linkage in the combined winding of phases A and B is

therefore equal to

$$\psi_{ab} = \psi_a - \psi_b \quad (\text{minus sign used because phase A is connected to phase B inverted})$$

$$\begin{aligned} &= -\frac{\chi_o}{3}(i\cos t - i\cos t) - \frac{\chi_d'' + \chi_q''}{3}\left(i\cos t + \frac{i\cos t}{2}\right) \\ &\quad - \frac{\chi_d'' - \chi_q''}{3}\left[i\cos t \cos 2\theta_o - i\cos t \cos\left(2\theta_o - \frac{2\pi}{3}\right)\right] \\ &\quad + \frac{\chi_o}{3}(-i\cos t + i\cos t) + \frac{\chi_d'' + \chi_q''}{3}\left(-i\cos t - \frac{i\cos t}{2}\right) \\ &\quad + \frac{\chi_d'' - \chi_q''}{3}\left[-i\cos t \cos\left(2\theta_o + \frac{2\pi}{3}\right) + i\cos t \cos\left(2\theta_o - \frac{2\pi}{3}\right)\right] \\ &= -(\chi_d'' + \chi_q'') i\cos t \\ &\quad - \frac{\chi_d'' - \chi_q''}{3}\left[i\cos t \cos 2\theta_o + i\cos t \cos\left(2\theta_o + \frac{2\pi}{3}\right)\right. \\ &\quad \left.- 2i\cos t \cos\left(2\theta_o - \frac{2\pi}{3}\right)\right] \\ &= -(\chi_d'' + \chi_q'') i\cos t \\ &\quad + (\chi_d'' - \chi_q'') i\cos t \cos\left(2\theta_o - \frac{2\pi}{3}\right) \end{aligned}$$

From Fig. 4.8, when the direct axis coincides with the axis of phases A and B combined,

$$\theta_o = -\frac{\pi}{6}$$

$$\begin{aligned} \therefore \psi_{ab} &= -(\chi_d'' + \chi_q'') i\cos t + (\chi_d'' - \chi_q'') i\cos t \cos(-\pi) \\ &= -(\chi_d'' + \chi_q'') i\cos t - (\chi_d'' - \chi_q'') i\cos t \\ &= -2\chi_d'' i\cos t \end{aligned}$$

$$\therefore \chi_{sd} = 2\chi_d'' \quad (4.10)$$

When the quadrature axis coincides with the axis of phases A and B combined

$$\theta_o = -\frac{2\pi}{3}$$

$$\begin{aligned}\therefore \psi_{ab} &= -(\chi_d'' + \chi_q'') i \cos t + (\chi_d'' - \chi_q'') i \cos t \cos(-2\pi) \\ &= -(\chi_d'' + \chi_q'') i \cos t + (\chi_d'' - \chi_q'') i \cos t \\ &= -2 \chi_q'' i \cos t\end{aligned}$$

$$\therefore \chi_{sq} = 2 \chi_q'' \quad (4.11)$$

$$\therefore \chi_{sd} = \frac{\chi_o + 2 \chi_d''}{3}, \quad \chi_{sq} = \frac{\chi_o + 2 \chi_q''}{3},$$

$$\text{And } \chi_{sd} = 2 \chi_d'', \quad \chi_{sq} = 2 \chi_q'',$$

$$\therefore \chi_o = 3 \chi_{sd} - \chi_{sd} = 3 \chi_{sq} - \chi_{sq}, \quad (4.12)$$

$$\chi_d'' = \frac{1}{2} \chi_{sd}, \quad (4.13)$$

$$\chi_q'' = \frac{1}{2} \chi_{sq}. \quad (4.14)$$

Therefore the values of χ_o , χ_d'' , and χ_q'' can be found from the static reactances which can be easily determined by test.

If the damper bars are removed or open-circuited during the test,

$$\chi_{sd} = 2 \chi_d', \quad \chi_{sq} = 2 \chi_q'.$$

If the field winding is also removed or open-circuited,

$$\chi_{sd} = 2 \chi_d, \quad \chi_{sq} = 2 \chi_q.$$

So the synchronous reactances and transient reactances can also be found from the static reactances.

4. 10. TIME-CONSTANTS

The time-constant of a circuit is the time required for a transient direct current to decay to $\frac{1}{e}$ or 0.368 of its initial value.

In a circuit of resistance r and reactance X the time constant is

$$T = \frac{L}{r} \text{ in seconds, or}$$

$$T = \frac{X}{r} = \frac{2\pi f L}{r} \text{ in radians.}$$

If the armature is open-circuited, the effective field-circuit time-constant is that of the field circuit alone, if the effect of the additional rotor circuits is neglected.

Let X_{ffd} = total field reactance
 R_{ffd} = total field resistance

Then the open-circuit field time constant is

$$T_{do}' = \frac{X_{ffd}}{2\pi f R_{ffd}} \quad (4.15)$$

If the armature is short-circuited, a transient current in field will induce a transient current in the armature, which will effect the apparent reactance of the field circuit.

From the elementary equivalent circuit, if we neglect the armature resistance which is usually very small in comparison with reactances, we find the apparent field transient reactance is

$$\begin{aligned} X_{ffd}' &= X_{ffd} - X_{afd} + \frac{X_l X_{afd}}{X_l + X_{afd}} \\ &= \frac{X_{ffd} X_d - X_{afd} X_d + X_l X_{afd}}{X_d} \\ &= \frac{X_{ffd} X_d - X_{afd} (X_d - X_l)}{X_d} \\ &= \frac{X_{ffd} X_d - X_{afd}^2}{X_d} \\ &= X_{ffd} \cdot \frac{X_d'}{X_d} \end{aligned}$$

So the short-circuit field time constant is

$$\begin{aligned} T_d' &= \frac{X_{ffd}'}{2\pi f R_{ffd}} \\ &= \left(\frac{X_d'}{X_d} \right) T_{do}' \end{aligned} \quad (4.16)$$

If there is an external reactance X_e in the armature circuit, it can be shown in the same way that the short-circuit field time constant is

$$T_d' = \left(\frac{X_d' + X_e}{X_d + X_e} \right) T_{do}' \quad (4.17)$$

If the effect of additional rotor circuits is not neglected, the time constants will have much more complicated expressions, and may be approximately determined from operational impedance equations or from equivalent circuits.

The open-circuit sub-transient time constant is the time constant of the additional rotor circuits considered as a single equivalent circuit when the armature is open-circuited, and the short-circuit sub-transient time constant is that when the armature is short-circuited. These are also very complicated; they may be determined from operational impedance equations or equivalent circuits.

It can be shown that

$$T_d'' = \left(\frac{X_d''}{X_d'} \right) T_{do}'' \quad (4.18)$$

In the quadrature axis, as there is no field circuit,

$$T_q'' = \left(\frac{\chi_q''}{\chi_q} \right) T_{q0}'' \quad (4.19)$$

In the armature circuit, since the field and the additional rotor circuits are always closed, the sub-transient reactances χ_d'' and χ_q'' must be used in the armature time constant. As the armature is at synchronous speed with respect to the rotor, the mean value of χ_d'' and χ_q'' is the apparent reactance for the time constant.

$$\therefore T_a = \frac{\frac{\chi_d'' + \chi_q''}{2}}{2\pi f r_a} = \frac{\chi_z}{2\pi f r_a} \quad (4.20)$$

When r_a is the d.c. resistance of the armature.

If there is external reactance χ_e ,

$$T_a = \frac{\chi_z + \chi_e}{2\pi f r_a} \quad (4.21)$$

Knowing these time constants, we can calculate the transient currents and voltages at any instant after the transient conditions have been applied.

The value of the transient reactances found from tests are usually unsaturated values, while in actual operation the magnetic paths are all saturated and therefore saturated values of reactances are wanted. Wright (36) determined from tests an empirical coefficient $F_{st} = 0.88$ which as a multiplier for the unsaturated reactances will give the saturated values:

$$\chi_d' = 0.88 \chi_{du}'$$

$$T_{du}' = \frac{\chi_{du}'}{\chi_d} T_{do}'$$

$$\therefore T_d' = \frac{\chi_d'}{\chi_d} T_{do}' = 0.88 \frac{\chi_{du}'}{\chi_d} T_{do}' = 0.88 T_{du}' \quad (4.22)$$

CHAPTER V

THE PER-UNIT SYSTEM

5. 1. PER-UNIT IMPEDANCES

For the sake of convenience, simplicity and clarity, the per-unit value system has been widely adopted. But it was not until the publication of Rankin's paper on per-unit impedances in 1945 that it was realised the per-unit impedance magnitudes depend upon the selection of the stator/rotor turn-ratio, which has no unique value. For instance, a turn-ratio could be defined as the ratio of the series turns per stator phase to the rotor turns per pole; an alternative turn-ratio could use the effective series turns per stator phase including the $k_p k_d$ factor; still another ratio could use the effective turns per pole appropriate to the fundamental components of the stator- and rotor-excited flux waves; and there are still other turn ratios, for all of which strong arguments have been advanced. Therefore while the various impedances employed by any one investigator may be mutually consistent, values given by different investigators do not always agree among themselves because of a different choice of effective turn-ratio.

It was pointed out by Rankin that the selection of turn-ratio is tantamount to the selection of unit rotor circuit current. As normal stator voltage is universally taken as unity, the product $X_{afd} I_{fd}$ must be unity at normal open-circuit airgap line stator voltage which shows that the unit field current I_{fd} is dependent on the choice of turn-ratio.

It was proved by Rankin that no unique value of base-current ratio is demanded by the general per-unit system. The base rotor-circuit currents are entirely susceptible to free selection, conditioned only by the restriction that when the value of rotor-circuit current has been selected as a base it must be used in the calculation of the machine impedances in the consistent method defined by the formulae. This means that the numerical values of many of the per-unit impedances are dependent upon the value of the rotor-circuit current which has been selected as a base. It was proved that all impedances measurable from the stator terminals such as X_d , X_d' , X_2 , r_a , etc., are independent of the values of rotor-circuit currents selected as bases. The impedances which are not invariant are those internal impedances which cannot be measured from the machine terminals.

5. 2. RANKIN'S CONVERSION FORMULAE

By converting the synchronous machine equations in the physical ampere-inch system into per-unit equations, Rankin derived per-unit impedance formulae in terms of the ampere-inch impedances and the selected base current ratios. The ampere-inch impedances are expressed in physical units (amperes, inches, seconds), while

the per-unit impedances are numerics. The base-current ratios are defined as the ratios of the rotor-circuit currents selected as bases to $1\frac{1}{2}$ times the peak value of the rated stator phase current.

The only condition imposed in the derivation of the conversion formulae is that the per-unit mutual impedances must be reciprocal. The formulae are:-

$$X_d = \frac{L_d}{L_{ao}} \quad (5.1)$$

$$X_{ad} = \frac{L_{ad}}{L_{ao}} \quad (5.2)$$

$$X_{afd} = X_{fad} = \frac{3}{2} \frac{L_{afd}}{L_{ao}} \left(\frac{I_{fdo}}{\frac{3}{2} i_{ao}} \right) \quad (5.3)$$

$$X_{and} = X_{nad} = \frac{3}{2} \frac{L_{and}}{L_{ao}} \left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right) \quad (5.4)$$

$$X_{ffd} = \frac{3}{2} \frac{L_{ffd}}{L_{ao}} \left(\frac{I_{fdo}}{\frac{3}{2} i_{ao}} \right)^2 \quad (5.5)$$

$$X_{nnd} = \frac{3}{2} \frac{L_{nnd}}{L_{ao}} \left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right)^2 \quad (5.6)$$

$$X_{ntd} = X_{fnd} = \frac{3}{2} \frac{L_{ntd}}{L_{ao}} \left(\frac{I_{fdo} I_{xdo}}{\frac{9}{4} i_{ao}^2} \right) \quad (5.7)$$

$$X_{nkd} = X_{knd} = \frac{3}{2} \frac{L_{nkd}}{L_{ao}} \left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right)^2 \quad (5.8)$$

$$r_a = \frac{r_a}{X_{ao}} \quad (5.9)$$

$$R_{ffd} = \frac{3}{2} \frac{R_{ffd}}{X_{ao}} \left(\frac{I_{fdo}}{\frac{3}{2} i_{ao}} \right)^2 \quad (5.10)$$

$$R_{nnd} = \frac{3}{2} \frac{R_{nnd}}{X_{ao}} \left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right)^2 \quad (5.11)$$

$$R_{ntd} = R_{fnd} = \frac{3}{2} \frac{R_{ntd}}{X_{ao}} \left(\frac{I_{fdo} I_{xdo}}{\frac{9}{4} i_{ao}^2} \right) \quad (5.12)$$

$$R_{nkd} = R_{kud} = \frac{3}{2} \frac{R_{nkd}}{X_{ao}} \left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right)^2 \quad (5.13)$$

The per-unit values of the rotor-circuit voltages are given by the following formulae:-

$$E_{fd} = \frac{E_{fd}}{e_{ao}} \left(\frac{I_{fdo}}{\frac{3}{2} i_{ao}} \right) \quad (5.14)$$

$$E_{nd} = \frac{E_{nd}}{e_{ao}} \left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right) \quad (5.15)$$

$$p = \frac{1}{\omega} p \quad (5.16)$$

In the above formulae, the bold-face type refers to physical ampere-inch quantities.

The subscript notation is as follows:-

- a , armature circuit
- f , field circuit
- k, n, x , additional rotor circuits
- d , direct-axis
- o , base quantities
- ff, nn , referring to self inductances and impedances
- af, fa, nf, fn, nk, kn , referring to mutual inductances and impedances between circuits denoted by the two letters

i_{ao} is the peak value of rated armature current, and e_{ao} is the peak value of rated voltage.

The direct-axis field-winding current as obtained in any per-unit system defined by the foregoing formulae will be in per-unit of the selected base current I_{fd0} , but the currents of all the additional rotor circuits are expressed in per-unit of the base current of the X^{th} direct-axis additional rotor circuit, with the ~~X^{th} direct-axis additional rotor circuit, with the~~ circuit submissive to free selection.

Only the direct-axis impedances are given explicitly in the preceding formulae. The quadrature-axis impedances can be obtained from the direct-axis expressions by merely replacing the d subscripts by q ; this will express all the quadrature-axis currents in terms of the selected quadrature-axis base currents. It is often advantageous to express the quadrature-axis currents in terms of the direct-axis bases since the latter are more easily determined. In this case the quadrature-axis per-unit impedances are obtained from the direct-axis formulae by first replacing all the d subscripts by q , and then replacing the quadrature-axis current bases equal to the direct-axis bases.

5. 3. SPECIFIC BASE-CURRENT RATIOS

The more important base-current ratios having practical advantages are given by the following four definitions. These base-current ratios cover nearly all in current use in the technical literature.

5. 3. (1) X_{ad} BASE

Base field current I_{fd0} is that current which will induce in each stator phase a voltage equal to $X_{ad} i_{a0}$; base rotor-circuit current I_{xd0} is similarly defined. These X_{ad} bases make X_{afd} and X_{axd} numerically equal to X_{ad} .

5. 3. (2) M.M.F. BASE

Base field current I_{fd0} is that which will give a m.m.f. per pole equal to the flat-topped armature reaction at normal stator current; base current I_{xd0} is similarly defined.

5. 3. (3) UNIT-VOLTAGE BASE

Base current I_{fd0} is the field current required to produce normal, open-circuit, airgap-line stator voltage; base current I_{xd0} is similarly defined.

5. 3. (4) EQUAL-MUTUAL BASE

The base rotor currents is defined so that the mutual impedances (X_{afd} , X_{axd} , X_{fxd}) for a machine with one

additional rotor circuit are all equal.

5. 4. ADOPTION OF X_{ad} BASE

Rankin suggested the X_{ad} bases as preferable for the calculation of machine impedances. There appears to be some agreement among designers on this point, and they have been adopted here.

The base field current I_{fdo} is defined in (5.3.1.) as that current which will induce in each stator phase a voltage equal to $X_{ad} i_{ao}$.

The base has the following advantages:-

(a) It makes X_{afd} numerically equal to X_{ad} , and avoids any further distinction between these quantities.

(b) It makes the stator leakage reactance ($X_d - X_{afd}$) identical with the more familiar "leakage" reactance ($X_d - X_{ad}$). With a similar definition of the base current in the fictitious quadrature-axis field circuit, the stator leakage reactance is the same for both axes.

(c) It is in more common use in recent literature than any other base and many of the impedance formulae are founded on it.

With three-phase currents of magnitude i_{ao} in the stator, and with the rotor circuits open, the maximum of the fundamental component flux density is as given by the following equation:-

$$B_{ga(max)} = 0.4\pi \frac{4}{\pi} \cdot \frac{3 i_{ao} k_p k_d n 2.54}{2 g} C_d, \quad (5.17)$$

The corresponding phase voltage is $X_{ad} i_{ao}$. By definition, I_{fdo} is the field current which will give a fundamental-component density equal to equation (5.17). The maximum of the fundamental component due to I_{fdo} is as given by the following equation.

$$B_{gf(max)} = 0.4\pi N_{fd} I_{fdo} \frac{2.54}{g} C_i, \quad (5.18)$$

The base-current ratio is thus given as:-

$$\frac{I_{fdo}}{\frac{3}{2} i_{ao}} = \frac{4}{\pi} \frac{C_d k_p k_d n}{C_i N_{fd}} \quad (5.19)$$

For a salient-pole machine with damper bars, the unit current for the additional rotor circuits is defined as that current which when flowing in the additional rotor circuit of 100 per cent pitch will induce in each stator phase a voltage $X_{ad} i_{ao}$. Let this current be I_{xdo} . The corresponding base-current ratio is given in the following equation:-

$$\frac{I_{xdo}}{\frac{3}{2} i_{ao}} = \frac{4}{\pi} \frac{C_d}{D_{dx}} \frac{k_p k_a n}{1} \quad (5.20)$$

where D_{dx} is defined as the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by the x^{th} additional rotor circuit in the direct axis.

A 100 per cent pitch circuit is not usually present in modern synchronous machines, but it is convenient to use it as a base circuit since it has maximum effectiveness.

5. 5. BASE VALUES

When per-unit values are used, all the quantities are expressed in per unit of the base values (unit values). These base values are as follows:-

E_{ao} = peak value of rated phase voltage

i_{ao} = peak value of rated stator line current

ψ_{ao} = peak value of rated stator phase linkages

L_{ao} = base (stator) inductance

$$= \frac{10^{-8} \psi_{ao}}{i_{ao}} = \frac{X_{ao}}{\omega}$$

X_{ao} = base reactance = ωL_{ao}

I_{fd0} = base value of field current

I_{xdo} = base value of current in additional rotor circuits.

E_{fd0} = base value of field voltage

$$= \frac{E_{ao}}{\left(\frac{I_{fd0}}{\frac{3}{2} i_{ao}} \right)}$$

E_{xdo} = base value of voltage in additional rotor circuits

$$= \frac{E_{ao}}{\left(\frac{I_{xdo}}{\frac{3}{2} i_{ao}} \right)}$$

$\bar{\psi}_{fd0}$ = base value of field linkage

$$= \frac{10^8 E_{fd0}}{\omega}$$

Ψ_{xdo} = base value of additional rotor circuit linkage

$$= \frac{10^8 E_{xdo}}{\omega}$$

Then the quantities to be expressed in per-unit of the above base values are shown in the following table.

e_a, e_b, e_c	in per-unit of	e_{ao}
i_d, i_q	"	i_{ao}
ψ_d, ψ_q	"	ψ_{ao}
E_{fd}	"	E_{fd0}
E_{nd}, E_{nq}	"	E_{xdo}
I_{fd}	"	I_{fd0}
I_{nd}, I_{nq}	"	I_{xdo}
Ψ_{fd}	"	Ψ_{fd0}
Ψ_{nd}, Ψ_{nq}	"	Ψ_{xdo}

C H A P T E R V I

EQUATIONS AND EQUIVALENT CIRCUITS IN THE PER-UNIT SYSTEM

6. 1. PER-UNIT FLUX LINKAGE AND VOLTAGE EQUATIONS OF IDEAL SYNCHRONOUS MACHINE

The linkage equations were derived as follows:-

$$\begin{aligned} \psi_d &= X_{afd} I_{fd} + X_{aid} I_{1d} + X_{azd} I_{2d} + \dots \\ &\quad - X_d i_d \end{aligned} \quad (6.1)$$

$$\begin{aligned} \psi_q &= X_{aq} I_{1q} + X_{azq} I_{2q} + \dots \\ &\quad - X_q i_q \end{aligned} \quad (6.2)$$

$$\begin{aligned} \bar{\Psi}_{fd} &= X_{ffd} I_{fd} + X_{fid} I_{1d} + X_{f2d} I_{2d} + \dots \\ &\quad - X_{fad} i_d \end{aligned} \quad (6.3)$$

$$\begin{aligned} \bar{\Psi}_{1d} &= X_{1fd} I_{fd} + X_{11d} I_{1d} + X_{12d} I_{2d} + \dots \\ &\quad - X_{1ad} i_d \end{aligned} \quad (6.4)$$

$$\begin{aligned} \bar{\Psi}_{1q} &= X_{11q} I_{1q} + X_{12q} I_{2q} + \dots \\ &\quad - X_{1aq} i_q \end{aligned} \quad (6.5)$$

$$\begin{aligned} \bar{\Psi}_{2d} &= X_{2fd} I_{fd} + X_{21d} I_{1d} + X_{22d} I_{2d} + \dots \\ &\quad - X_{2ad} i_d \end{aligned} \quad (6.4a)$$

$$\begin{aligned} \bar{\Psi}_{2q} &= X_{21q} I_{1q} + X_{22q} I_{2q} + \dots \\ &\quad - X_{2aq} i_q \end{aligned} \quad (6.5a)$$

.....

The rotor-circuit voltage equations are as follows:-

$$\begin{aligned} E_{fd} &= p \bar{\Psi}_{fd} + R_{ffd} I_{fd} + R_{fid} I_{1d} + R_{f2d} I_{2d} \\ &\quad + \dots \end{aligned} \quad (6.6)$$

$$E_{1d} = p \Phi_{1d} + R_{1fd} I_{fd} + R_{11d} I_{1d} + \dots \quad (6.7)$$

$$E_{1q} = p \Phi_{1q} + R_{11q} I_{1q} + R_{12q} I_{2q} + \dots \quad (6.8)$$

$$E_{2d} = p \Phi_{2d} + R_{2fd} I_{fd} + R_{21d} I_{1d} + \dots \quad (6.7a)$$

$$E_{2q} = p \Phi_{2q} + R_{21q} I_{1q} + R_{22q} I_{2q} + \dots \quad (6.8a)$$

As the damper bars are all short-circuited, there is no voltage in the additional rotor circuits

$$\therefore E_{1d} = E_{2d} = \dots = E_{1q} = E_{2q} = \dots = 0 \quad (6.9)$$

6. 2. OPERATIONAL IMPEDANCES

By solving the rotor-circuit voltage equations for the rotor-circuit currents, I_{fd} , I_{1d} , I_{2d} , \dots , I_{1q} , I_{2q} , \dots and substituting in the armature linkage equations, the armature flux linkage ψ_d and ψ_q can be expressed in the following form.

$$\psi_d = G(p) E_{fd} - \chi_d(p) i_d \quad (6.10)$$

$$\psi_q = -\chi_q(p) i_q \quad (6.11)$$

where $G(p)$, $\chi_d(p)$ and $\chi_q(p)$ are operational functions, or functions of the operator p with coefficients composed exclusively of impedances. Therefore $G(p)$, $\chi_d(p)$ and $\chi_q(p)$ are termed operational impedances of the machine.

As by Heaviside's operational calculus, when $t = 0$, $p = \infty$,

and when $t = \infty$, $p = 0$, it is only necessary to put p equal to ∞ or 0 in order to find $\chi_d(p)$ and $\chi_q(p)$ at $t = 0$ or $t = \infty$. By definition, the impedance at $t = 0$ is the sub-transient impedance and the impedance at $t = \infty$ is the steady state impedance.

Therefore neglecting the resistances,

$$\begin{aligned}\chi_d'' &= \chi_d(\infty) \\ \chi_q'' &= \chi_q(\infty)\end{aligned}\quad (6.12)$$

$$\begin{aligned}\chi_d &= \chi_d(0) \\ \chi_q &= \chi_q(0)\end{aligned}\quad (6.13)$$

For illustration, we consider the case of no additional rotor circuit and that of one additional rotor circuit in each axis.

6. 2. (1) NO ADDITIONAL ROTOR CIRCUIT

In the direct-axis,

$$\begin{aligned}E_{fd} &= p \mathcal{I}_{fd} + R_{ffd} I_{fd} \\ &= p (\mathcal{X}_{ffd} I_{fd} - \mathcal{X}_{afd} i_d) + R_{ffd} I_{fd}\end{aligned}$$

$$\therefore I_{fd} = \frac{E_{fd} + p \mathcal{X}_{afd} i_d}{p \mathcal{X}_{ffd} + R_{ffd}}$$

$$\begin{aligned}\psi_d &= G(p) E_{fd} - \chi_d(p) i_d \\ &= \mathcal{X}_{afd} I_{fd} - \chi_d i_d \\ &= \frac{\mathcal{X}_{afd} E_{fd} + p \mathcal{X}_{afd}^2 i_d - p \mathcal{X}_{ffd} \chi_d i_d - R_{ffd} \chi_d i_d}{p \mathcal{X}_{ffd} + R_{ffd}} \\ &= \frac{\mathcal{X}_{afd}}{p \mathcal{X}_{ffd} + R_{ffd}} E_{fd} - \frac{p (\mathcal{X}_{ffd} \chi_d - \mathcal{X}_{afd}^2) + R_{ffd} \chi_d}{p \mathcal{X}_{ffd} + R_{ffd}} i_d\end{aligned}\quad (6.14)$$

$$\therefore G(p) = \frac{\mathcal{X}_{afd}}{p \mathcal{X}_{ffd} + R_{ffd}}$$

$$X_d(p) = \frac{p(\Sigma_{ffd} X_d - \Sigma_{afd}^2) + R_{fd} X_d}{p \Sigma_{ffd} + R_{fd}} \quad (6.15)$$

When $p = \infty$,

$$\begin{aligned} X_d(\infty) &= X_d' = \frac{\Sigma_{ffd} X_d - \Sigma_{afd}^2}{\Sigma_{ffd}} \\ &= X_d - \frac{\Sigma_{afd}^2}{\Sigma_{ffd}} \end{aligned} \quad (6.16)$$

In the quadrature axis,

$$\psi_q = -X_q(p) i_q = -X_q i_q$$

$$\therefore X_q(p) = X_q \quad (6.17)$$

When $p = \infty$,

$$\begin{aligned} X_q(\infty) &= X_q' \\ &= X_q \end{aligned} \quad (6.18)$$

6. 2. (2) ONE ADDITIONAL ROTOR CIRCUIT IN EACH AXIS

In the direct axis,

$$\begin{aligned} E_{fd} &= p \Phi_{fd} + R_{fd} I_{fd} \\ &= p(\Sigma_{ffd} I_{fd} + \Sigma_{fid} I_{fd} - \Sigma_{fad} i_d) + R_{fd} I_{fd} \end{aligned}$$

$$\begin{aligned} E_{ld} = 0 &= p \Phi_{ld} + R_{ld} I_{ld} \\ &= p(\Sigma_{lfd} I_{fd} + \Sigma_{lld} I_{ld} - \Sigma_{lad} i_d) + R_{ld} I_{ld} \end{aligned}$$

Solving for I_{fd} and I_{ld} ,

$$I_{fd} = \frac{(p \Sigma_{ld} + R_{ld}) E_{fd} + [p^2(\Sigma_{ld} \Sigma_{afd} - \Sigma_{fld} \Sigma_{ald}) + p \Sigma_{afd} R_{ld}] i_d}{p^2(\Sigma_{ld} \Sigma_{ffd} - \Sigma_{fld}^2) + p(\Sigma_{ld} R_{fd} + \Sigma_{ffd} R_{ld}) + R_{ld} R_{fd}}$$

$$I_{ld} = \frac{-p \Sigma_{ld} E_{fd} + [p^2(\Sigma_{ld} \Sigma_{ald} - \Sigma_{fld} \Sigma_{afd}) + p \Sigma_{ald} R_{fd}] i_d}{p^2(\Sigma_{ld} \Sigma_{ffd} - \Sigma_{fld}^2) + p(\Sigma_{ld} R_{fd} + \Sigma_{ffd} R_{ld}) + R_{ld} R_{fd}}$$

Substituting the values of I_{fd} and I_{ld} into the armature flux linkage equation,

$$\begin{aligned}\psi_d &= G(p) E_{fd} - X_d(p) i_d \\ &= \Sigma_{afd} I_{fd} + \Sigma_{ald} I_{ld} - X_d i_d \\ &= \frac{p(\Sigma_{lld} \Sigma_{afd} - \Sigma_{fld} \Sigma_{ald}) + \Sigma_{afd} R_{lld}}{A(p)} E_{fd} \\ &\quad - \left[X_d - \frac{p^2(\Sigma_{lld} \Sigma_{afd}^2 - 2\Sigma_{fld} \Sigma_{ald} \Sigma_{afd} + \Sigma_{ffd} \Sigma_{ald}^2) + p(\Sigma_{afd}^2 R_{lld} + \Sigma_{ald}^2 R_{ffd})}{A(p)} \right] i_d\end{aligned}$$

where $A(p) = p^2(\Sigma_{lld} \Sigma_{ffd} - \Sigma_{fld}^2) + p(\Sigma_{lld} R_{ffd} + \Sigma_{ffd} R_{lld}) + R_{lld} R_{ffd}$

When $p = \infty$,

$$X_d(\infty) = X_d''$$

$$= X_d - \frac{\Sigma_{lld} \Sigma_{afd}^2 - 2\Sigma_{fld} \Sigma_{ald} \Sigma_{afd} + \Sigma_{ffd} \Sigma_{ald}^2}{\Sigma_{lld} \Sigma_{ffd} - \Sigma_{fld}^2} \quad (6.19)$$

When $\Sigma_{lld} = \infty$, it is equivalent to the case of no additional rotor circuit.

Then

$$X_d(p) = X_d - \frac{\Sigma_{afd}^2}{\Sigma_{ffd}} = X_d'$$

i.e., the impedance reduces to the transient reactance.

In the quadrature axis,

$$\begin{aligned}E_{lq} = 0 &= p \Psi_{lq} + R_{llq} I_{lq} \\ &= p(\Sigma_{llq} I_{lq} - \Sigma_{laq} i_q) + R_{llq} I_{lq}\end{aligned}$$

$$\therefore I_{lq} = \frac{\Sigma_{laq} i_q}{p \Sigma_{llq} + R_{llq}} \quad - 46 -$$

$$\begin{aligned}
 \psi_q &= -\chi_q(p) \dot{i}_q \\
 &= \Sigma_{1aq} I_{1q} - \chi_q \dot{i}_q \\
 &= -\left[\chi_q - \frac{p \Sigma_{1aq}^2}{p \Sigma_{11q} + R_{11q}} \right] \dot{i}_q \\
 \therefore \chi_q(p) &= \chi_q - \frac{p \Sigma_{1aq}^2}{p \Sigma_{11q} + R_{11q}}
 \end{aligned}$$

(6.20)

When $p = \infty$,

$$\chi_q(\infty) = \chi_q'' = \chi_q - \frac{\Sigma_{1aq}^2}{\Sigma_{11q}}$$

(6.21)

$G(p)$, $\chi_d(p)$, and $\chi_q(p)$ can be found in similar manner for any number of additional rotor circuits and therefrom the subtransient reactances. The results are cumbersome and are too complicated for practical use.

6. 3. EQUIVALENT CIRCUITS OF THE SYNCHRONOUS MACHINE

Analagously with the transformer, a synchronous machine can be represented by equivalent circuits. Two such circuits are needed for a synchronous machine, one for the direct- and another for the quadrature-axis, because the circuits in the two axes differ by reason of (a) the saliency of poles and (b) the field winding existing only in the direct-axis. The equivalent circuit method of analysis is very effective in the solution of machine and system problems involving a number of simultaneous equations. In the case of the synchronous machine the complete equivalent circuit (complete in the sense that the field-winding circuit and the multiple damper-winding circuits are individually included) is very useful whenever a detailed knowledge is needed of the operation of all the rotor circuits. Specific examples of its use are in the design of the damper-windings, in problems involving single-phase and synchronous operation, in the determination of damping and synchronizing torque and in the determination of transient and sub-transient impedances.

Linville presented an equivalent circuit which is complete within the limits of certain well-defined approximations, and also derived formulae for all the machine impedances. Liwschitz developed simplified equivalent circuits which are easier to use and give satisfactory results in many important problems.

But in those applications which require a knowledge of the details of damper-winding operation the complete equivalent circuits are indispensable. Rankin consequently derived more complete and exact equivalent circuits, primarily for use on A.C.

network analyzers*.

6. 4. RANKIN'S EQUIVALENT CIRCUITS

Rankin's equivalent circuits were developed directly from the operational equations of the synchronous machine by noting the physical relations which exist between the various impedances.

Now consider the D-axis circuit only. The generalized per-unit equations are as follows:-

$$\psi_d = X_{afd} I_{fd} + X_{aid} I_{1d} + X_{ard} I_{2d} + \dots - X_d i_d$$

$$\bar{\Psi}_{fd} = X_{ffd} I_{fd} + X_{fid} I_{1d} + X_{f2d} I_{2d} + \dots - X_{fad} i_d$$

$$\bar{\Psi}_{1d} = X_{1fd} I_{fd} + X_{11d} I_{1d} + X_{12d} I_{2d} + \dots - X_{1ad} i_d$$

$$\bar{\Psi}_{2d} = X_{2fd} I_{fd} + X_{21d} I_{1d} + X_{22d} I_{2d} + \dots - X_{2ad} i_d$$

$$E_{fd} = p \bar{\Psi}_{fd} + R_{ffd} I_{fd} + R_{fid} I_{1d} + R_{f2d} I_{2d} + \dots$$

$$E_{1d} = p \bar{\Psi}_{1d} + R_{1fd} I_{fd} + R_{11d} I_{1d} + R_{12d} I_{2d} + \dots$$

$$E_{2d} = p \bar{\Psi}_{2d} + R_{2fd} I_{fd} + R_{21d} I_{1d} + R_{22d} I_{2d} + \dots$$

If all the rotor-circuit voltages are zero, the foregoing equations reduce to:-

$$\psi_d = X_{afd} I_{fd} + X_{aid} I_{1d} + X_{ard} I_{2d} + \dots - X_d i_d \quad (6.22)$$

$$0 = \left(X_{ffd} + \frac{R_{ffd}}{p} \right) I_{fd} + \left(X_{fid} + \frac{R_{fid}}{p} \right) I_{1d} + \left(X_{f2d} + \frac{R_{f2d}}{p} \right) I_{2d} + \dots - X_{fad} i_d \quad (6.23)$$

$$0 = \left(X_{1fd} + \frac{R_{1fd}}{p} \right) I_{fd} + \left(X_{11d} + \frac{R_{11d}}{p} \right) I_{1d} + \left(X_{12d} + \frac{R_{12d}}{p} \right) I_{2d} + \dots - X_{1ad} i_d \quad (6.24)$$

* In actual analysers the residual resistance in inductive reactors is usually excessive for the purpose intended. Rankin suggested advancing the phase-angle of the impedance components, so that resistance is represented by capacitors, and inductive reactance by resistors.

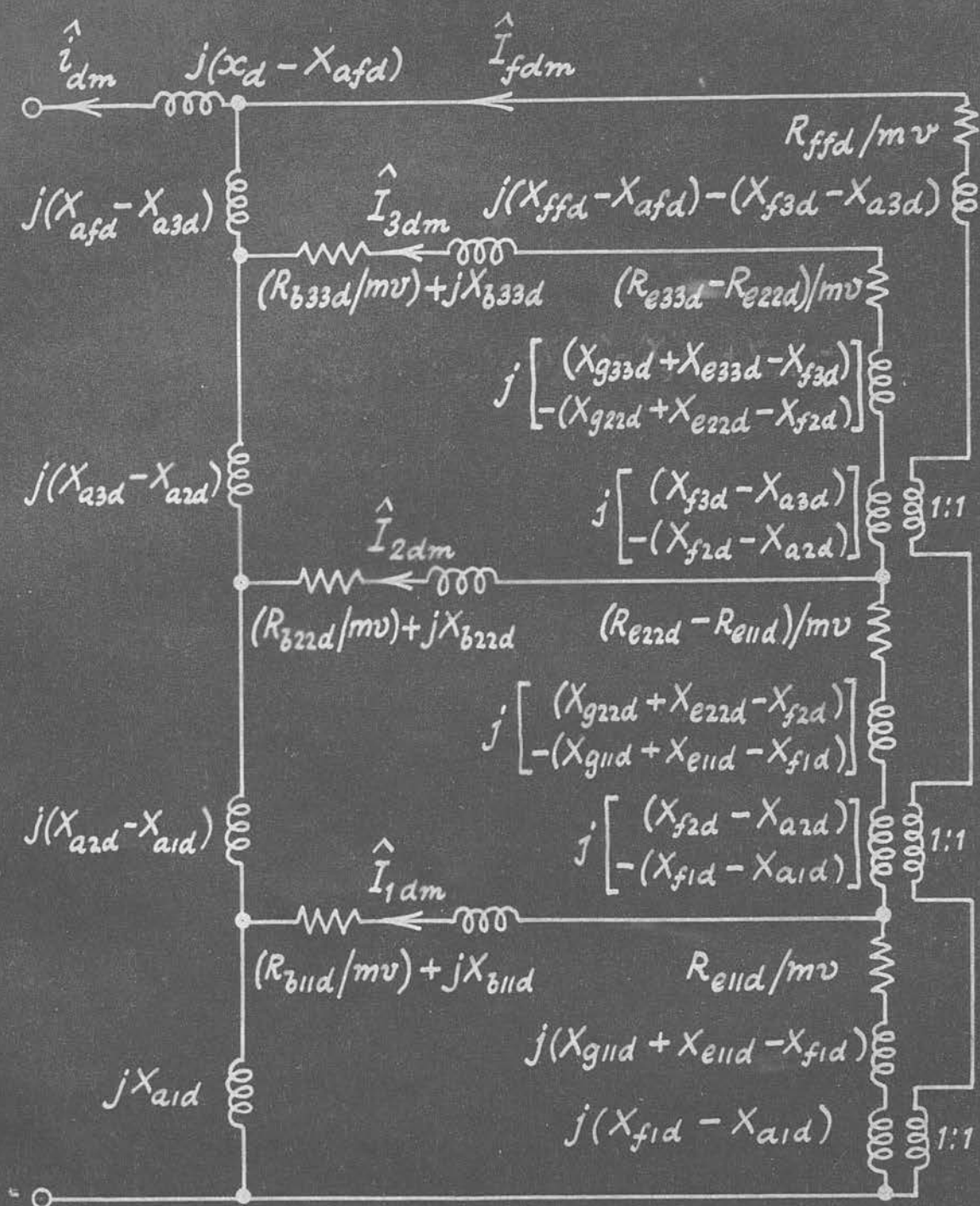


FIG. 6.1. Direct-axis complete equivalent circuit for five or six bars per pole.

R/mv = resistance ; jX = inductive reactance.

$$0 = \left(X_{zfd} + \frac{R_{zfd}}{p} \right) I_{fd} + \left(X_{zld} + \frac{R_{zld}}{p} \right) I_{ld} + \left(X_{z2d} + \frac{R_{z2d}}{p} \right) I_{2d} + \dots - X_{zad} i_d \quad (6.24a)$$

It is evident from the above that ψ_d and all the rotor currents are linear operational functions of i_d . If i_d is assumed to have the vectorial form $i_d = \hat{i}_{dm} e^{+jmvt}$, then ψ_d and the rotor currents must have the vector forms:-
 $\psi_d = \hat{\psi}_{dm} e^{+jmvt}$; $I_{fd} = \hat{I}_{fdm} e^{+jmvt}$; $I_{ld} = \hat{I}_{ldm} e^{+jmvt}$
 The mv notation is used because it is adaptable to different machine operating conditions. The m specifies the order of the harmonic when any are present, and the v is a generalized rotor-velocity term. For a synchronous operation at constant slip s , and with only fundamental currents flowing, v is replaced by s , and m equals unity only and may be dropped. For asynchronous single-phase operation, harmonics are present, and m is needed to define the particular harmonic being studied, and v becomes equal to the rotor velocity.

Substituting the vectorial forms of i_d , ψ_d , I_{fd} , and I_{ld} into the above equations, using only the steady-state solution of the operational equations and cancelling the exponentials, gives the following equations:-

$$\hat{\psi}_{dm} = X_{afd} \hat{I}_{fdm} + X_{ald} \hat{I}_{ldm} + X_{a2d} \hat{I}_{2dm} + \dots - X_d \hat{i}_{dm} \quad (6.25)$$

$$0 = \left(X_{ffd} + \frac{R_{ffd}}{jm v} \right) \hat{I}_{fdm} + \left(X_{fld} + \frac{R_{fld}}{jm v} \right) \hat{I}_{ldm} + \left(X_{f2d} + \frac{R_{f2d}}{jm v} \right) \hat{I}_{2dm} + \dots - X_{fad} \hat{i}_{dm} \quad (6.26)$$

$$0 = \left(X_{lfd} + \frac{R_{lfd}}{jm v} \right) \hat{I}_{fdm} + \left(X_{lld} + \frac{R_{lld}}{jm v} \right) \hat{I}_{ldm} + \left(X_{l2d} + \frac{R_{l2d}}{jm v} \right) \hat{I}_{2dm} + \dots - X_{lad} \hat{i}_{dm} \quad (6.27)$$

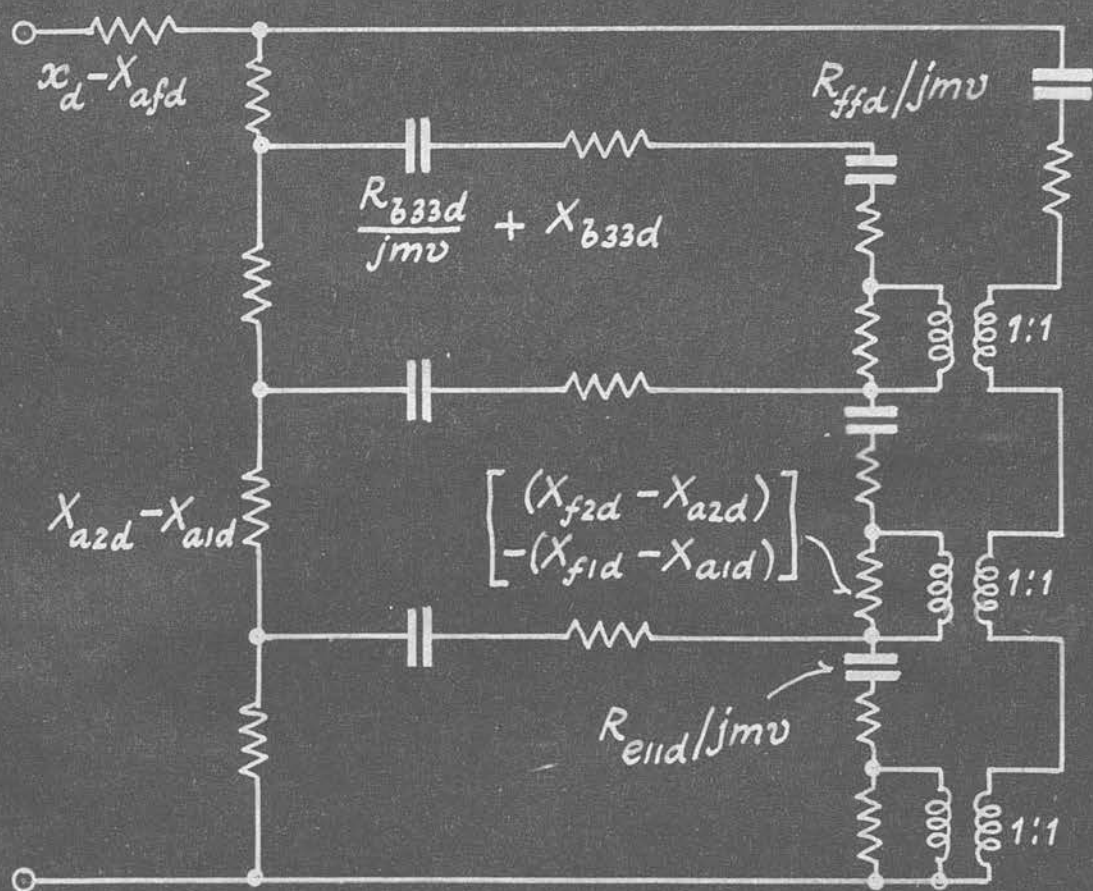


FIG. 6.2. Direct-axis equivalent circuit corresponding to Fig. 6.1.

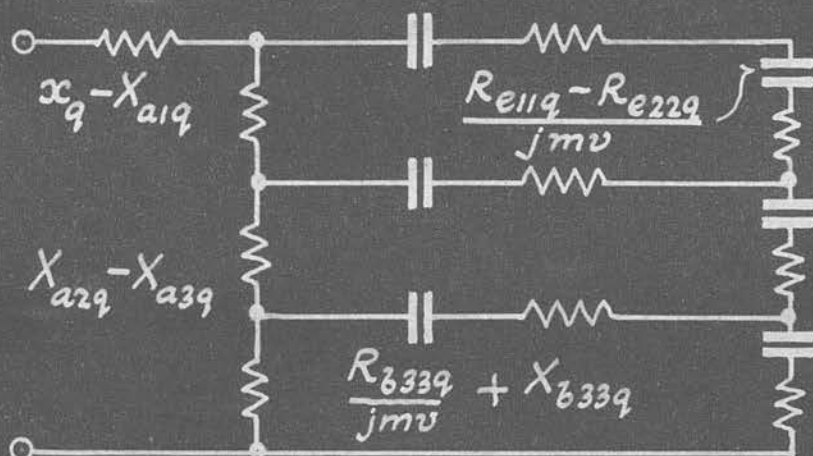


FIG. 6.4. Quadrature-axis equivalent circuit corresponding to Fig. 6.3.

The circuits above represent those of Figs. 6.1 and 6.3 with all components divided by j . Typical components are shown. $R/j\omega$ = capacitive reactance; X = resistance.

$$0 = (\Sigma_{2fd} + \frac{R_{2fd}}{j\omega\mu}) \hat{I}_{fdm} + (\Sigma_{21d} + \frac{R_{21d}}{j\omega\mu}) \hat{I}_{1dm} \\ + (\Sigma_{22d} + \frac{R_{22d}}{j\omega\mu}) \hat{I}_{2dm} + \dots - \Sigma_{2ad} \hat{I}_{adm} \quad (6.27a)$$

The above can be applied to the subject problem by recognizing the physical relations which exist between the self- and mutual impedances of the various rotor circuits.

Consider the n th additional rotor circuit. The reactance X_{nnd} is the sum of the reactance due to the airgap flux within the bars which form the n th additional rotor circuit, the reactance due to the leakage flux in the bar slots and the reactance due to the end-ring flux.

$$X_{nnd} = X_{gnnd} + X_{bnnd} + X_{ennd} \quad (6.28)$$

The mutual reactance X_{nkd} between the n th additional rotor circuit and any outer additional rotor circuit k is the sum of the reactance due to the airgap flux and the reactance due to the end-ring flux; the bar-slot flux is pure leakage.

$$X_{nkd} = X_{gnnd} + X_{ennd} \quad (k > n) \quad (6.29)$$

The mutual reactance between the n th additional rotor circuit and any inner additional rotor circuit is obtainable from the inner-circuit reactances. This important relation is a direct result of the reciprocal per-unit mutual impedances.

The mutual reactance between the n th additional rotor circuit and the field-winding circuit depends only on the airgap flux of the n th additional rotor circuit, since the bar-slot flux and the end-ring flux are not mutual with the field-winding. These mutual reactances are reciprocal, too.

Concerning the resistance components, the resistance of the n th additional rotor circuit is the sum of the bar and end-ring resistances of that circuit.

$$R_{nnd} = R_{bnnd} + R_{ennd} \quad (6.30)$$

The mutual resistance between the n th additional rotor circuit and any other additional rotor circuit k is the end-ring resistance

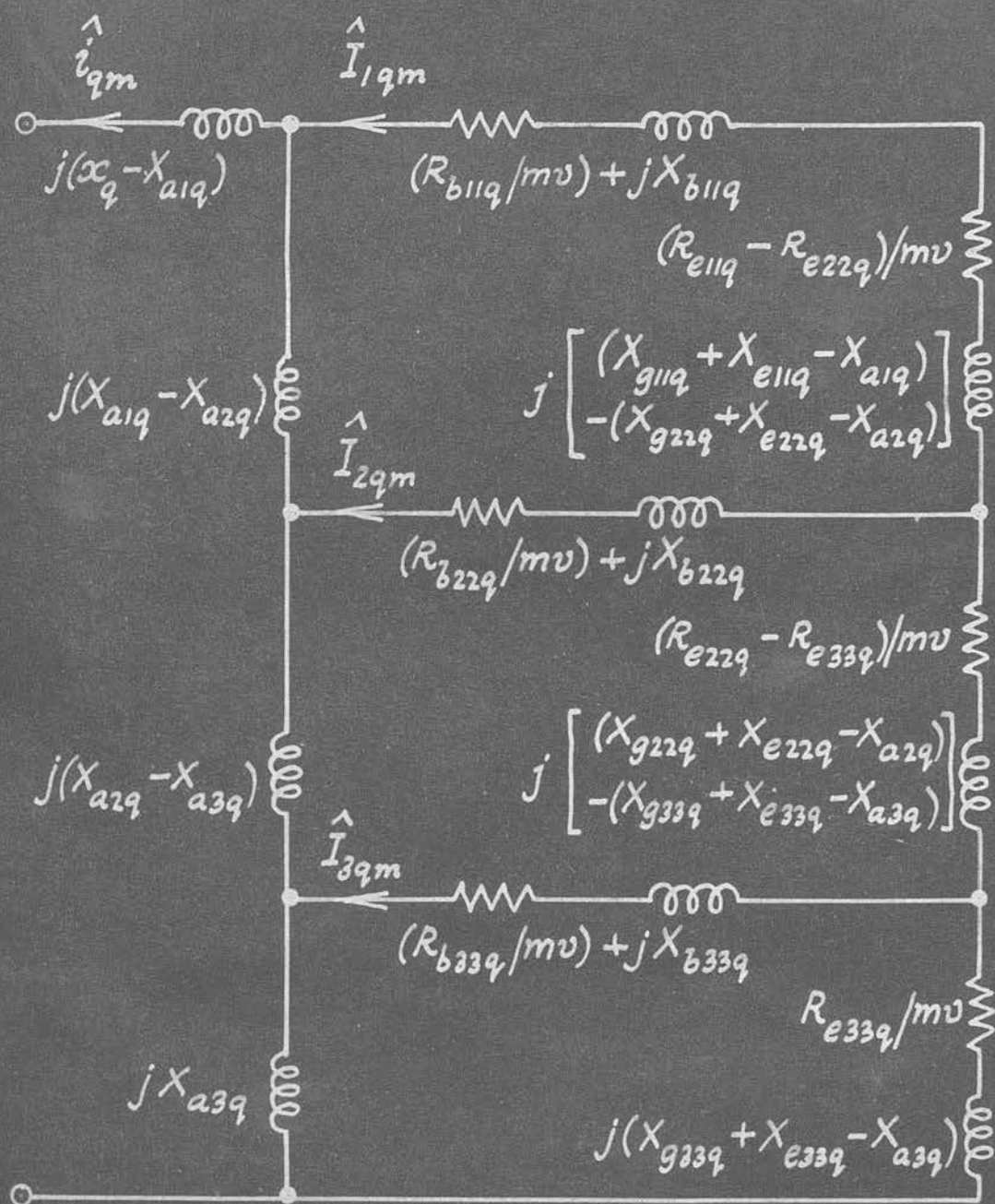


FIG. 6.3. Quadrature-axis complete equivalent circuit for five or six bars per pole.

R/mv = resistance; jX = inductive reactance.

of the n^{th} circuit, since the bar resistance is not mutual with the k^{th} circuit.

$$R_{nkd} = R_{eund} \quad (k > n) \quad (6.31)$$

The mutual resistance between the n^{th} circuit and any inner-circuit is obtained from the inner-circuit resistances as these mutual impedances are reciprocal.

There are no mutual resistances between the additional rotor circuits and the field-winding circuits since these circuits are coupled only magnetically.

The direct-axis equivalent circuit can then be obtained as follows. The numbering of the damper-bar circuits proceeds outward from the polar axis. If a single bar lies directly on the axis, it should be hypothetically divided at its centre line with the halves thus becoming bar 1 on each side of the polar axis.

The end-ring impedance is correctly represented and is separated from the field-winding circuit by means of $1/2$ coupling transformers, Fig. 6.1. Fig. 6.2. shows the modification used for an actual equivalent-circuit set-up to avoid the resistance inherent in inductive reactor components.

The quadrature-axis equivalent circuit is similar to the direct-axis one except that there is no field winding circuit. It is shown in Fig. 6.3, while Fig. 6.4 shows the more practicable equivalent circuit network for actual investigation.



C H A P T E R V I I

CALCULATION OF SYNCHRONOUS MACHINE CONSTANTS

7. 1. GENERAL

In the previous three chapters the various synchronous machine constants, their per-unit values, the equivalent circuits, etc., have been discussed. Methods ^{and formulae} for the calculation of these machine constants are now considered. Several methods of calculation have been given for these constants: Doherty and Shirley, Alger, and Kilgore derived formulae for armature leakage reactances; Linville, Kilgore, and Rankin derived expressions for nearly all the other constants. Rankin developed the most complete equivalent circuits and obtained systematic formulae for all the impedances appearing therein. Knowing these impedances, the transient and sub-transient reactances can be determined from the equivalent circuits. Rankin's equivalent circuits were introduced in the preceding chapter and his methods of calculation of the impedances are adopted here. Those constants not given in Rankin's paper are adopted from Alger and Kilgore. Doherty and Shirley's leakage reactances are not so correctly expressed and have been superseded, while Linville's formulae are based on his own equivalent circuits, which are not so complete as Rankin's.

7. 2. RANKIN'S METHOD

Rankin's method of calculating the per-unit impedances is first to determine the ampere-inch-second values and convert them to per-unit values by the conversion factors given in the preceding chapter, eq. (5.1)-(5.16).

The X_{ad} base is used for the selection of base rotor currents. Therefore unit field current I_{fd0} is that which will induce in each stator phase a voltage of $X_{ad} i_{a0}$. This base current has the advantage that it makes X_{afd} numerically equal to X_{ad} and so avoids a distinction between these two quantities. The corresponding base-current ratio is given by

$$\frac{I_{fd0}}{\frac{3}{2} i_{a0}} = \frac{4}{\pi} \cdot \frac{C_d}{C_1} \frac{k_p k_a n}{N_{fd}} \quad (7.1)$$

The unit current for the additional rotor circuits I_{x0} is that which, when flowing in the additional rotor circuit of 100 per cent pitch, will induce in each stator phase a voltage $X_{ad} i_{a0}$. A 100-per-cent-pitch circuit is not usually present in modern synchronous machines, but it is convenient to use it as a base circuit since it has maximum effectiveness. The corresponding

base-current ratio is given by

$$\frac{I_{xdo}}{\frac{3}{2} i_{ao}} = \frac{4}{\pi} \frac{C_d}{D_{dix}} \frac{k_p k_a n}{1} \quad (7.2)$$

where D_{dix} is the ratio of the maximum of the fundamental component of flux density to the maximum density when the machine is excited by the x^{th} additional rotor circuit in the direct-axis.

Base stator inductance L_{ao} and fundamental flux per pole at rated voltage will be needed for the evaluation of the per-unit impedances. These quantities are expressed in the following in terms of machine dimensions.

The maximum flux density of field at no load and rated voltage is:-

$$B_{max.} = 3.19 \frac{F_g}{g} \quad (7.3)$$

where F_g is the field ampere turns per pole for rated voltage and g is the minimum effective gap.

The maximum fundamental flux density is:-

$$B_1 = 3.19 \frac{F_g}{g} C_1 \quad (7.4)$$

The flux per pole due to field is:-

$$\Phi_f = 12.76 \frac{R_a l}{P_1 g} F_g C_1 \quad (7.5)$$

where R_a = armature radius

l = machine stacked length

P_1 = number of poles

Let A = flat-topped armature reaction at base stator current

$$= 1.5 i_{ao} k_p k_a n$$

Then

$$\frac{1}{L_{ao}} = \frac{2\pi f i_{ao}}{E_{ao}} \quad (7.6)$$

$$\begin{aligned}
 \text{Or } \frac{1}{Z_{ao}} &= 10^8 \frac{I_{ao}}{\Psi_{ao}} = \frac{10^8}{1.5 P_1} \left(\frac{1}{K_p K_a n} \right)^2 \frac{A}{\Phi_f} \\
 &= \frac{10^8}{19.14} \frac{A}{F_g C_1 R_a l} \left(\frac{1}{K_p K_a n} \right)^2 \quad (7.7)
 \end{aligned}$$

7.3. POLE-SHAPE COEFFICIENTS

The calculation of synchronous machine impedances involves some pole-shape coefficients: D_{don} , D_{qon} , D_{din} , D_{qin} , C_d , C_q , C_1 , and K_f . These must be evaluated by flux plots if extreme accuracy is desired, since the actual magnetic gap obviously cannot be represented exactly by any known mathematical expression. Alternatively the coefficients may be evaluated by numerical integration of the gap permeance, which is approximately expressed by formula in terms of the pole dimensions. The dimensions of the pole of a salient-pole machine is shown in Fig. 7.1, where y_d and y_q are measured in per unit of half the pole pitch (0.5τ). y_{dn} and y_{qn} are the values of y_d and y_q , respectively, to circuit n , measured from the corresponding direct- or quadrature-axis.

α = ratio of pole arc to pole pitch.

ρ = ratio of maximum gap to minimum gap.

The following equations were given by Rankin for the expression of the gap reluctance. These expressions give the gap length in per unit of the minimum gap g .

$$g_y = g_d \text{ in the region } 0 < y_d < \alpha \tau \quad (7.8)$$

$$g_y = g_q \text{ in the region } \alpha \tau < y_d < \tau \quad (7.9)$$

$$g_d = 1 + (\rho - 1) \left(\frac{y_d}{\alpha} \right)^2 \quad (7.10)$$

$$g_q = g_\tau + \beta g_\tau \sin^2 \left(\frac{y_d - \alpha \tau}{1 - \alpha \tau} \right) \frac{\pi}{2} \quad (7.11)$$

$$g_\tau = 1 + (\rho - 1) \left(\frac{\alpha \tau}{\alpha} \right)^2 \quad (7.12)$$

$$\beta = -1 + \sqrt{1 + \frac{1}{4} \left(\frac{1 - \alpha}{\tau + g g_\tau} \right)^2} \quad (7.13)$$

$$\alpha \tau = \alpha - 3.5 \rho \tau^{-1} g \quad (7.14)$$

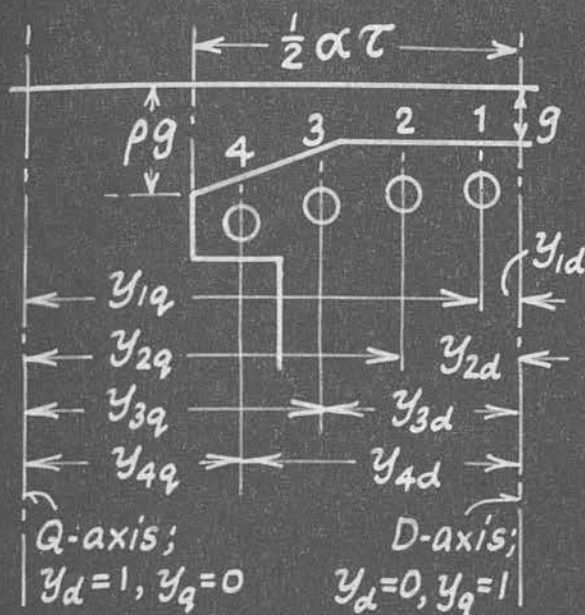


FIG. 7.1. Damper dimensions.

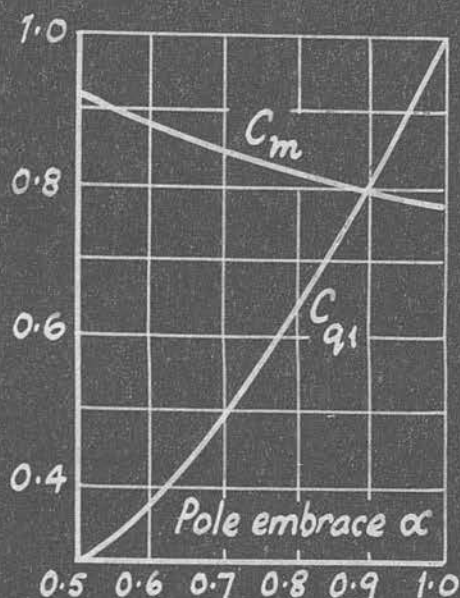


FIG. 7.2. Flux coefficients.

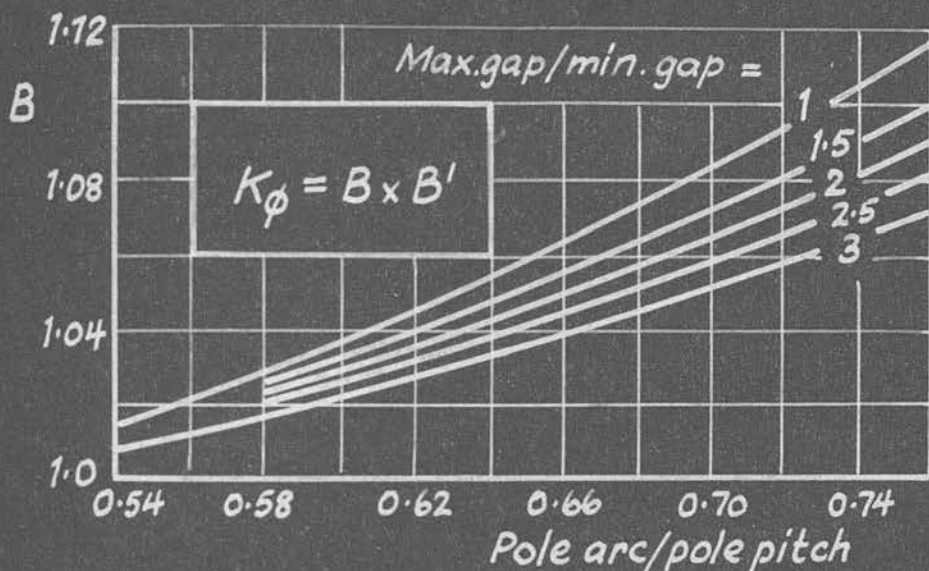
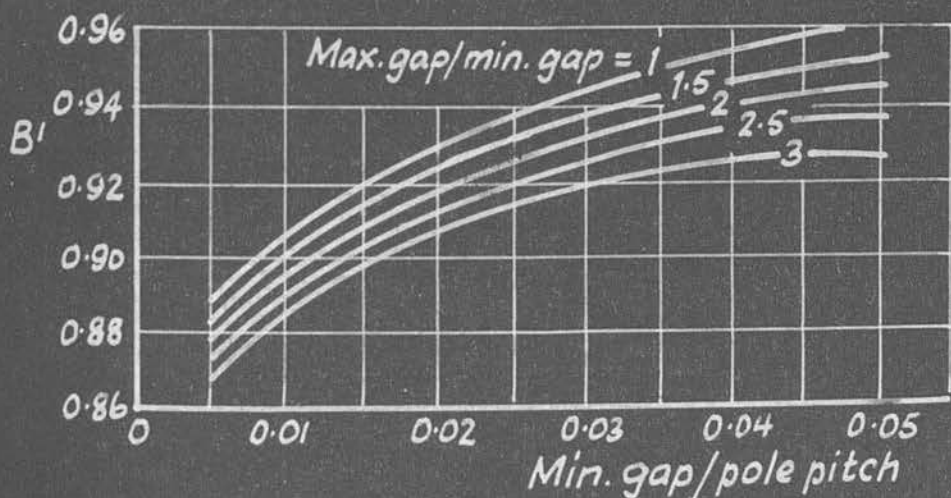


FIG. 7.3.

Flux
distrib-
ution
coeff-
icients.



$$y_d + y_q = 1.0 \quad (7.15)$$

7.3. (1) D_{don}

The factor $(\frac{2}{\pi}) D_{don}$ is defined as that by which the maximum gap density must be multiplied to obtain the average density within the damper circuit of span $2y_{nd}$ with the machine excited by the damper circuit of that span in the direct axis.

Flux per pole

$$\Phi = 2 \int_0^{y_{nd}} \frac{3.19}{g g_y} dy_d = 2 \frac{3.19}{g} \frac{2}{\pi} D_{don} y_{nd}$$

$$\therefore D_{don} y_{nd} = \frac{\pi}{2} \int_0^{y_{nd}} g_y^{-1} dy_d \quad (7.16)$$

7.3. (2) D_{qon}

The factor $(\frac{2}{\pi}) D_{qon}$ is analogous to $(\frac{2}{\pi}) D_{don}$, but refers to the quadrature axis:-

$$D_{qon} y_{nq} = \frac{\pi}{2} \int_0^{y_{nq}} g_y^{-1} dy_q = \frac{\pi}{2} \int_{1-y_{nq}}^1 g_y^{-1} dy_d \quad (7.17)$$

7.3. (3) D_{din}

This is defined as the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by the n^{th} additional rotor circuit in the direct axis.

Flux density

$$B_y = \frac{3.19}{g g_y}$$

$$B_{y \text{ fund.}} = 2 \int_0^{y_{nd}} \frac{3.19}{g g_y} \cos \frac{\pi}{2} y_d dy_d = \frac{3.19}{g} D_{din}$$

$$\therefore D_{din} = 2 \int_0^{y_{nd}} g_y^{-1} \cos \frac{\pi}{2} y_d dy_d \quad (7.18)$$

7.3. (4) D_{q1n}

D_{q1n} is analogous to D_{d1n} , but in the quadrature axis.

$$\begin{aligned} D_{q1n} &= 2 \int_0^{y_{4q}} g_y^{-1} \cos \frac{\pi}{2} y_q dy_q \\ &= 2 \int_{1-y_{4q}}^1 g_y^{-1} \sin \frac{\pi}{2} y_d dy_d \end{aligned} \quad (7.19)$$

7.3. (5) C_{d1}

C_{d1} is defined as the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by a sine-wave armature m.m.f. in the direct axis.

$$B_y = \frac{3.19}{9g_y} \cos \frac{\pi}{2} y_d$$

$$B_{y \text{ fund.}} = 2 \int_0^1 \frac{3.19}{9g_y} \cos^2 \frac{\pi}{2} y_d dy_d = \frac{3.19}{9} C_{d1}$$

$$\therefore C_{d1} = 2 \int_0^1 g_y^{-1} \cos^2 \frac{\pi}{2} y_d dy_d \quad (7.20)$$

Kilgore derived an approximate expression for a coefficient C_m , which is the ratio of C_{d1} to C_1 , so that if C_1 is known, C_{d1} can be found from it and the expression for C_m . This expression is much simpler than Rankin's expression for C_{d1} , and as pointed out by Kilgore it checks quite satisfactorily with the value of C_{d1} obtained from Wieseman's curves. The expression for C_m is:-

$$C_m = \frac{\alpha \pi + \sin \alpha \pi}{4 \sin \frac{\alpha \pi}{2}}$$

where α is the pole embrace.

C_m was plotted against α by Kilgore and is reproduced here in Fig. 7.2.

7.3. (6) C_{q1}

C_{q1} is analogous to C_{d1} but in the quadrature axis.

$$\begin{aligned} C_{q1} &= 2 \int_0^1 g_y^{-1} \cos^2 \frac{\pi}{2} y_d dy_d \\ &= 2 \int_0^1 g_y^{-1} \sin^2 \frac{\pi}{2} y_d dy_d \end{aligned} \quad (7.21)$$

Kilgore also derived an approximate expression for C_{q1} which is:-

$$C_{q1} = \frac{4\alpha + 1}{5} - \frac{\sin \alpha \pi}{\pi}$$

A curve for C_{q1} was also plotted against α and is reproduced in Fig. 7.2.

7.3. (7) C_1

C_1 is defined as the factor by which the maximum flux density must be multiplied to obtain the peak of the fundamental component of flux density with the machine excited by the direct-axis field winding.

The field winding links only the flux which enters the pole, and its effective span or pitch is accordingly somewhat less than 100 per cent. However, the effective span is greater than the physical pole arc, because the field winding links the flux which enters the side of the pole tip. Rankin showed that relatively little error is introduced by assuming that C_1 is given by

$$D_{din} \text{ for } y_{ud} = 1.00.$$

7.3. (8) K_ϕ

K_ϕ is the factor by which the total fundamental flux per pole must be multiplied to obtain the total flux per pole, with the machine excited by the direct-axis field winding.

By means of the assumption that the field winding circuit is equivalent to a damper-winding circuit of 100 per cent pitch, K_ϕ can be evaluated in terms of D_{don} and D_{din} , for $y_{ud} = 1.00$.

$$\text{Total flux per pole} = \frac{3.19}{9} \frac{2}{\pi} D_{don}$$

$$\text{Fundamental flux per pole} = \frac{3.19}{9} \frac{2}{\pi} D_{din}$$

$$\therefore K_\phi = \frac{D_{don}}{D_{din}} \bigg|_{y_{ud} = 1.0} \quad (7.22)$$

7.3. (9)

Wieseman presented curves, based on actual flux plots, for the quantities C_d , C_q , C_i , and K_p . The curves for C_d , C_q , and C_i were reproduced in Chapter III, and the curves for K_p are given here in Fig. 7.3. Rankin showed a quite close agreement between numerical values of these four quantities obtained by numerical integration given above and the corresponding value obtained from Wieseman's curves for a typical pole configuration. This agreement may warrant the use of the integral expressions for the pole-shape coefficients D_{don} , D_{qon} , D_{din} , and D_{qin} until corrected values can be obtained from actual flux plots.

The integral expressions for C_d , C_q , C_i , and K_p therefore serve as a check for the reliability of the expressions for D_{don} , D_{qon} , D_{din} , and D_{qin} . In calculation of impedances, the values of C_d , C_q , C_i , and K_p are to be taken from Wieseman's curves, as they are the most satisfactory; while for the values D_{don} , D_{qon} , D_{din} , and D_{qin} , the integral expressions are resorted to, as no actual flux plots are available.

7.4. IMPEDANCES OF DIRECT-AXIS CIRCUITS

7.4. (1) X_{ad} , X_l , and X_d

The stator synchronous reactance X_d can be obtained by separating it into the components X_{ad} and $(X_d - X_{ad})$ or X_l . The former is the reactance of armature reaction, and the latter is the leakage reactance.

At the instant when the resulting armature m.m.f. wave is in the direct axis, the fundamental flux per pole will be as follows:-

Fundamental flux per pole

$$\Phi_a = \frac{3.19}{9} \frac{4}{\pi} 1.5 i_{ao} k_p k_a n C_d \frac{2}{\pi} \frac{2\pi R_a l}{P_i} \quad (7.23)$$

$$\begin{aligned} \therefore L_{ad} &= \frac{\Phi_a (k_p k_a n) P_i}{10^8 i_{ao}} \\ &= \frac{76.56}{\pi 10^8} \frac{(k_p k_a n)^2 C_d R_a l}{9} \end{aligned} \quad (7.24)$$

$$\therefore X_{ad} = \frac{L_{ad}}{L_{ao}} = \frac{4}{\pi} \frac{C_d}{C_i} \frac{A}{F_g} = k_{ad} \frac{A}{F_g} \quad (7.25)$$

Let the specific permeance (λ) be defined as the effective

flux per pole per inch of core length produced by unit ampere turns per pole. Then according to Kilgore, the specific permeance for armature leakage reactance can be separated into three parts λ_i , λ_e , and λ_o ,

where λ_i = specific permeance for the slot tooth-tip and zigzag leakage combined,

λ_e = specific permeance for the end-winding leakage reactance, and

λ_o = specific permeance for the belt leakage reactance.

Then by the definition of specific permeance, the total armature leakage inductance is

$$\begin{aligned} L_l &= \frac{4}{\pi} 1.5 (k_p k_a n) l (k_p k_a n) P_i 10^8 (\lambda_i + \lambda_e + \lambda_o) \\ &= \frac{6 \times 10^8}{\pi} (k_p k_a n)^2 P_i l (\lambda_i + \lambda_e + \lambda_o) \end{aligned} \quad (7.26)$$

$$\therefore X_l = \frac{L_l}{L_{a0}} = \frac{1}{3.19 \pi} \frac{A_g P_i}{F_g C_a R_a} (\lambda_i + \lambda_e + \lambda_o) \quad (7.27)$$

$$\therefore L_d = L_{ad} + L_l$$

$$= \frac{76.56}{\pi 10^8} (k_p k_a n)^2 l \left[\frac{R_a C_{d1}}{g} + \frac{P_i}{12.76} (\lambda_i + \lambda_e + \lambda_o) \right] \quad (7.28)$$

$$\therefore X_d = X_{ad} + X_l$$

$$= K_{ad} \left[\frac{A}{F_g} + \frac{A_g P_i}{12.76 F_g R_a C_{d1}} (\lambda_i + \lambda_e + \lambda_o) \right] \quad (7.29)$$

The formulae for λ_i , λ_e , and λ_o were derived by Kilgore as follows:-

$$\lambda_i = C_x \frac{20}{39} \left[\frac{h_2}{b_s} + \frac{h_1}{3b_s} + 0.2 + 0.07 \frac{b_c}{g'} \right] \quad (7.30)$$

for a two-layer winding three phase machine, where h_1 , h_2 ,

b_s , b_t are dimensions of the slot and are shown in the Fig. 7.4.

Here q = slots per phase per pole,
 g' = actual gap in pole centre.

$$C_x = \frac{k_x}{k_p^2 k_d^2}$$

$$k_x = \frac{1}{2} \left(1 + \cos \frac{n\pi}{3} \right) \quad (7.31)$$

at $\frac{3-n}{3}$ pitch, n being any integer.

The value of k_x was plotted by Alger and is reproduced here as Fig. 7.5; the value of C_x was plotted by Kilgore and is reproduced here as Fig. 7.6.

$$\lambda_B = 0 \text{ (negligible) for 3 phase machine} \quad (7.32)$$

$$\lambda_e = \frac{4}{l} (2l_{e2} + l_{e1}) \quad (7.33)$$

where l_{e1} = extension of bent section of end-winding,
 l_{e2} = length of straight section of end-winding.
 l_{e1} and l_{e2} are shown in Fig. 7.7.

7. 4. (2) FIELD-WINDING REACTANCE, \mathcal{L}_{fd}

With a current of one ampere flowing in the field winding, the flux per pole due to airgap flux is given by the following equation:-

$$\text{Flux per pole} = 3.19 \frac{N_{fd}}{g} \frac{2}{\pi} C_0 \frac{2\pi R_a l}{P_i} \quad (7.34)$$

where $\frac{2}{\pi} C_0$ is the ratio of average flux density to the maximum flux density.

$$\therefore \mathcal{L}_{fd} = 12.76 \times 10^{-8} \frac{R_a l}{g} N_{fd}^2 C_0 \quad (7.35)$$

Let λ_b = specific permeance for the pole-body leakage, and
 λ_t = specific permeance for the pole-tip leakage.

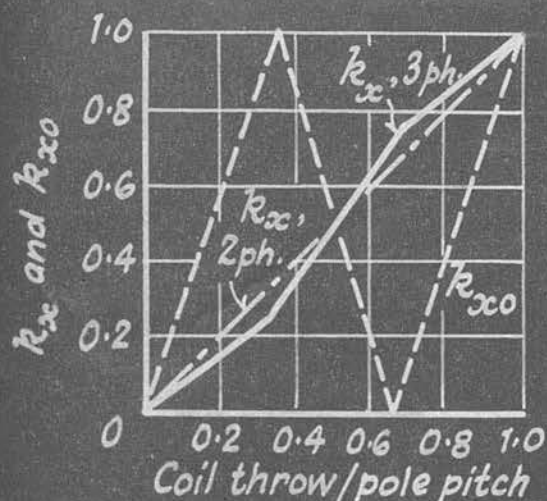


FIG. 7.5. Factors k_x and k_{xo}

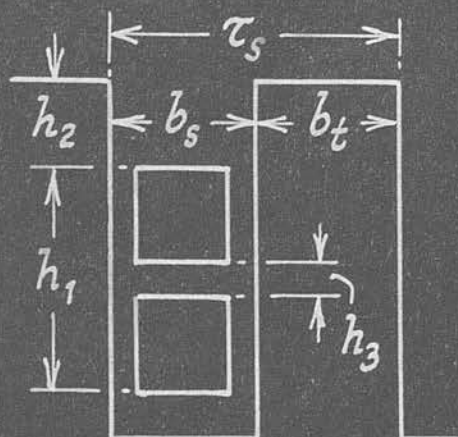


FIG. 7.4. Dimensions of slot for two-layer winding.

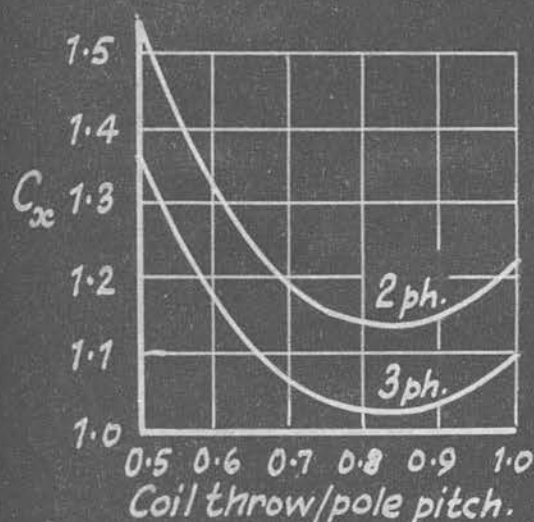


FIG. 7.6. Slot leakage factor.

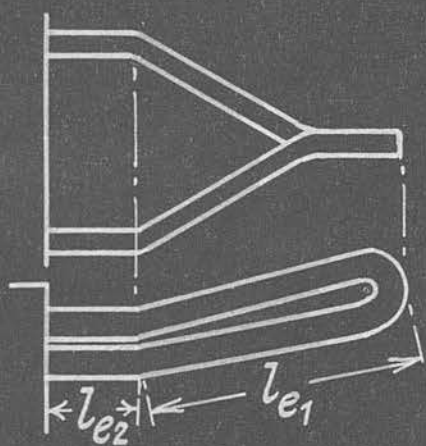


FIG. 7.7. Dimensions of end-winding.

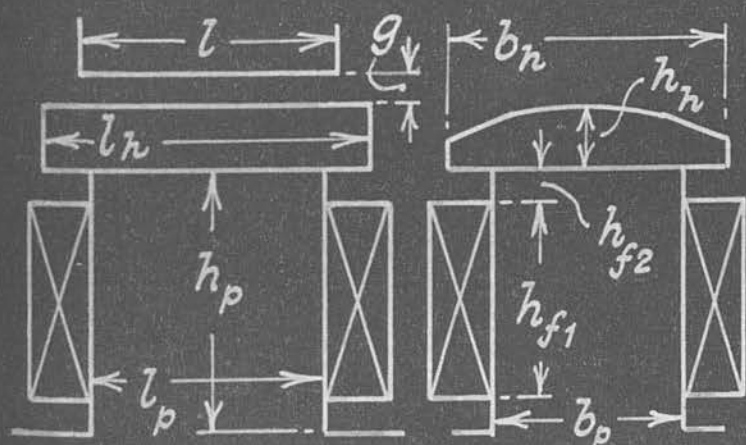


FIG. 7.8. Dimensions of pole.

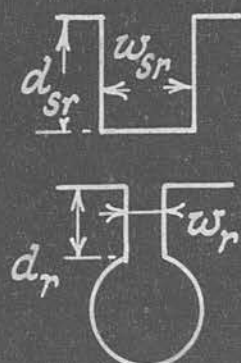


FIG. 7.9. Rotor slots.

Then the inductance in henrys of the field-winding circuit of P_i poles due to the pole-tip and pole-body flux is given by the following equation.

$$L_{ffd} = 10^{-8} N_{fd}^2 l P_i (\lambda_b + \lambda_t) \quad (7.36)$$

The total inductance in henrys of the field-winding circuit is given by

$$\begin{aligned} L_{ffd} &= 10^{-8} N_{fd}^2 P_i l \left[\frac{12.76 R_a C_0}{P_i g} + (\lambda_b + \lambda_t) \right] \\ &\approx 10^{-8} N_{fd}^2 P_i l \left[\frac{K_f \Phi_f}{F_g l} + (\lambda_b + \lambda_t) \right] \end{aligned} \quad (7.37)$$

$$\begin{aligned} \therefore X_{ffd} &= \frac{3}{2} \frac{L_{ffd}}{L_{ao}} \left(\frac{I_{fd0}}{\frac{3}{2} i_{ao}} \right)^2 \\ &= K_{ad} K_f X_{ad} + K_{ad}^2 \frac{A l}{\Phi_f} (\lambda_b + \lambda_t) \end{aligned} \quad (7.38)$$

The specific permeances λ_b and λ_t are difficult to evaluate and depend considerably upon the individual characteristics of a machine.

Kilgore derived formulae of λ_b and λ_t for a given pole shape, Fig. 7.8.

Let τ_r = pole pitch on rotor diameter

D_r = rotor diameter

Then, as derived by Kilgore,

$$\begin{aligned} \lambda_b &= 3.19 \times \frac{4}{3} \left[\frac{3(h_h + g - 0.055 \tau_r)}{(\tau_r - b_h)} \right. \\ &\quad \left. + \frac{h_{f1} + 3h_{f2} + 0.1 \tau_r (1 - \frac{10}{P_i})}{\frac{\pi}{P_i} (D_r - 2h_h - 0.4 h_{f1}) - b_p} \right] \end{aligned} \quad (7.39)$$

$$\lambda_t = 3.19 \left[\frac{4(l_h - l) + 2h_{f1} + 0.5 b_p}{l} \right] \quad (7.40)$$

7. 4. (3) MUTUAL REACTANCE BETWEEN STATOR AND FIELD-WINDING, X_{afd}

The fundamental flux per pole per ampere field-winding current is obtained as follows:-

Fundamental flux per pole

$$= 3.19 \frac{N_{fd}}{g} \frac{2}{\pi} C_1 \frac{2\pi R_a l}{P_1} \quad (7.41)$$

If sinusoidal distribution of the armature winding is assumed, the corresponding mutual inductance in henrys is given as follows:-

$$L_{afd} = 3.19 \times 10^{-8} \frac{N_{fd}}{g} C_1 4 R_a l k_p k_a n \quad (7.42)$$

$$\begin{aligned} \therefore X_{afd} &= \frac{3}{2} \frac{L_{afd}}{L_{ao}} \left(\frac{I_{fdo}}{\frac{3}{2} i_{ao}} \right) \\ &= \frac{4}{\pi} \frac{C_d}{C_1} \frac{A}{F_g} = k_{ad} \frac{A}{F_g} = X_{ad} \end{aligned} \quad (7.43)$$

7. 4. (4) ADDITIONAL-ROTOR-CIRCUIT REACTANCE FOR n th CIRCUIT, X_{nnd}

With one ampere in the n th additional rotor circuit, the average flux density in the airgap within the bars which bound the n th circuit is:-

$$B_{av.} = \frac{3.19}{g} \frac{2}{\pi} D_{don} \quad (7.44)$$

The corresponding inductance in henrys for the entire circuit of P_1 poles is:-

$$L_{gnnd} = 12.76 \times 10^{-8} \frac{R_a l}{g} D_{don} Y_{nd} \quad (7.45)$$

The inductance due to the bar-slot leakage flux is:-

$$L_{bnnd} = 6.38 \times 10^{-8} P_1 l \left(\frac{dr}{w_r} + 0.625 \right) \quad (7.46)$$

for open round slots. For rectangular slots, the factor 0.625 within the bracket should be replaced by $(0.333 \frac{d_{sr}}{w_{sr}})$; for closed slots, the bracketed expression should be replaced by test or estimated values of the slot permeance. The dimensions of the slots are shown in Fig. 7.8.

The inductance due to the end-ring leakage flux can be approximated by considering the end rings as two wires of a single-phase transmission line with a distance between centres of D_e and an (effective) cross-sectional radius of r_e . This inductance is given by the following equation.

$$L_{ennd} = 0.508 \times 10^{-8} P_{lend} \left(9.2 \log_{10} \frac{D_e}{r_e} + 1 \right) \quad (7.47)$$

$$\therefore X_{gnd} = \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} \frac{D_{don}}{C_1} Y_{nd} \quad (7.48)$$

$$X_{bund} = 0.5 \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} \frac{P_g}{C_1 R_a} \left(\frac{d_r}{w_r} + 0.625 \right) \quad (7.49)$$

$$X_{ennd} = 0.04 \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} \frac{P_g}{C_1 R_a} \frac{l_{end}}{l} \left(9.2 \log_{10} \frac{D_e}{r_e} + 1 \right) \quad (7.50)$$

$$\therefore X_{nnd} = X_{gnd} + X_{bund} + X_{ennd} \quad (7.51)$$

7. 4. (5) MUTUAL REACTANCE BETWEEN STATOR AND n th ADDITIONAL ROTOR CIRCUIT, X_{nad}

With one ampere flowing in the n th additional rotor circuit, the maximum value of the fundamental component of flux density is given as follows:-

$$B_{fund.} = \frac{3.19}{9} D_{din} \quad (7.52)$$

The mutual inductance in henrys between the stator and the additional rotor circuit is then:-

$$L_{nad} = 12.76 \times 10^{-8} \frac{R_a l}{9} k_p k_u n D_{din} \quad (7.53)$$

$$\therefore X_{nad} = \left(\frac{4}{\pi} \frac{C_d}{D_{dix}} \right) \frac{A}{F_g} \frac{D_{din}}{C_1} = \frac{D_{din}}{D_{dix}} X_{ad} \quad (7.54)$$

Note that X_{nad} becomes numerically equal to X_{ad} for the x th (100% pitch) additional rotor circuit.

7. 4. (6) MUTUAL REACTANCE BETWEEN ADDITIONAL ROTOR CIRCUITS, X_{ukd} ($k > n$)

As discussed in the preceding chapter, the mutual reactance between additional rotor circuits is equal to the airgap and ending reactances of the inner circuit.

$$L_{ukd} = L_{gnd} + L_{end} \quad (7.55)$$

$$X_{ukd} = X_{gnd} + X_{end} \quad (7.56)$$

7. 4. (7) MUTUAL REACTANCE BETWEEN FIELD-WINDING CIRCUIT AND n th ADDITIONAL ROTOR CIRCUIT, X_{ufd}

The mutual inductance L_{ufd} between the field-winding circuit and the n th additional rotor circuit is $N_{fd} L_{gnd}$, since the flux which defines the latter inductance is mutual with the field circuit.

$$\therefore L_{ufd} = N_{fd} L_{gnd} = 12.76 \times 10^{-8} \frac{R_{el}}{g} D_{don} Y_{nd} N_{fd} \quad (7.57)$$

$$X_{ufd} = \left(\frac{4}{\pi} \frac{C_d}{C_1} \right) \frac{D_{don}}{D_{dix}} Y_{nd} X_{ad} \quad (7.58)$$

7. 4. (8) STATOR RESISTANCE, r_a

The per-unit value is $r_a = \frac{r_a}{X_{ao}} \quad (7.59)$

7. 4. (9) FIELD-WINDING CIRCUIT RESISTANCE, R_{fd}

The per-unit value is

$$R_{fd} = \frac{10^8}{\omega P_1} \left(\frac{4}{\pi} \frac{C_d}{C_1} \right)^2 \frac{1}{N_{fd}^2} \frac{A}{\Phi_f} R_{fd} \quad (7.60)$$

7. 4. (10) RESISTANCE OF THE ADDITIONAL ROTOR CIRCUIT, R_{rud}

$$R_{rud} = 1.67 \times 10^{-6} \frac{l_{bn}}{a_{bn}} P_1 \quad (7.61)$$

$$R_{ennd} = 3.33 \times 10^{-6} \frac{l_{end}}{a_{end}} P_i \quad (7.62)$$

$$R_{nnd} = 1.67 \times 10^{-6} P_i \left(\frac{l_{bn}}{a_{bn}} + 2 \frac{l_{end}}{a_{end}} \right) \quad (7.63)$$

$$R_{nnd} = \frac{10^8}{\omega P_i} \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{\Phi_f} R_{nid} \quad (7.64)$$

7. 4. (11) MUTUAL RESISTANCE BETWEEN ADDITIONAL ROTOR CIRCUITS, $R_{nkd} \ (k > n)$

The mutual resistance between the n^{th} additional rotor circuit and any outer rotor circuit k is the end-ring resistance of the n^{th} additional rotor circuit.

$$R_{nkd} = R_{ennd} = 3.33 \times 10^{-6} \frac{l_{end}}{a_{end}} P_i \quad (7.65)$$

$$R_{nkd} = \frac{10^8}{\omega P_i} \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{\Phi_f} R_{nkd} \quad (7.66)$$

7. 5. IMPEDANCES OF QUADRATURE-AXIS CIRCUITS

The currents in the additional rotor circuits in the quadrature-axis are expressed in per-unit of I_{xdo} which is the base current of the direct-axis additional rotor circuits. This base is adopted so that the per-unit currents in the direct- and quadrature-axis additional rotor circuits may be added directly. Therefore the per-unit quadrature-axis impedances are obtained by substituting q for d in the corresponding direct-axis ampere-inch-second impedance formula, and converting the result into a per-unit value based on the base-current ratio (7.2).

7. 5. (1) X_{aq} AND X_q

X_q is evaluated by separating it into the two components X_{aq} and $(X_q - X_{aq})$ or X_l , analogously to the method used in the evaluation of X_d . $(X_q - X_{aq})$ is taken as independent of the rotor position and is equal to $(X_d - X_{ad})$.

$$X_{aq} = \frac{4}{\pi} \frac{C_{q1}}{C_1} \frac{A}{F_g} = \frac{C_{q1}}{C_{d1}} X_{ad} \quad (7.67)$$

$$X_l = X_q - X_{aq} = X_d - X_{ad} \quad (7.68)$$

$$X_q = X_{aq} + X_l \quad (7.69)$$

7. 5. (2) n th ADDITIONAL ROTOR-CIRCUIT REACTANCE, X_{nnq}

$$X_{gnnq} = \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} \frac{D_{gon}}{C_1} Y_{nq} \quad (7.70)$$

$$X_{bnnq} = 0.5 \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} \frac{P_1 g}{C_1 R_a} \left(\frac{dr}{w_r} + 0.625 \right) \quad (7.71)$$

$$X_{ennq} = 0.04 \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} \frac{P_1 g}{C_1 R_a} \frac{l_{enq}}{l} \left(9.2 \log_{10} \frac{D_e}{l_e} + 1 \right) \quad (7.72)$$

$$X_{nnq} = X_{gnnq} + X_{bnnq} + X_{ennq} \quad (7.73)$$

7. 5. (3) MUTUAL REACTANCE BETWEEN STATOR AND n th ADDITIONAL ROTOR CIRCUIT, X_{naq}

$$X_{naq} = \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right) \frac{A}{F_g} \frac{D_{gin}}{C_1} = \frac{D_{gin}}{D_{din}} X_{ad} \quad (7.74)$$

7. 5. (4) MUTUAL REACTANCE BETWEEN ADDITIONAL ROTOR CIRCUITS, X_{nkg} ($k > n$)

$$X_{nkg} = X_{gung} + X_{eng} \quad (7.75)$$

7. 5. (5) RESISTANCE OF n th ADDITIONAL ROTOR CIRCUIT, R_{nnq}

$$R_{nnq} = 1.67 \times 10^{-6} P_1 \left(\frac{l_{bn}}{a_{bn}} + 2 \frac{l_{enq}}{a_{enq}} \right) \quad (7.76)$$

$$R_{nnq} = \frac{10^8}{\omega P_1} \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{F_g} R_{nnq} \quad (7.77)$$

7. 5. (6) MUTUAL RESISTANCE BETWEEN ADDITIONAL ROTOR CIRCUITS, R_{nkg} ($k > n$)

$$R_{nkg} = R_{eng} = 3.33 \times 10^{-6} \frac{l_{enq}}{a_{enq}} P_1 \quad (7.78)$$

$$R_{nkq} = \frac{10^8}{\omega P_1} \left(\frac{4}{\pi} \frac{C_{d1}}{D_{dix}} \right)^2 \frac{A}{\Phi_f} R_{nkq} \quad (7.79)$$

7. 6. TRANSIENT AND SUB-TRANSIENT REACTANCES

The transient reactance can be found from the equivalent circuits or the operational impedances by neglecting the additional rotor circuits.

It was found in the preceding chapter that

$$X_d' = X_d - \frac{X_{afd}^2}{X_{Hd}} \quad (7.80)$$

As there is no field-winding circuit in the quadrature-axis

$$X_q' = X_q \quad (7.81)$$

The sub-transient reactances are equal to the operational impedances when $t = 0$, or $p = \infty$.

$$\therefore X_d'' = X_d(p) \quad p = \infty = X_d(0) \quad (7.82)$$

$$X_q'' = X_q(p) \quad p = \infty = X_q(0) \quad (7.83)$$

They are also equal to the resultant reactance of the whole direct-axis or quadrature-axis equivalent circuit respectively when all the resistances are equal to zero. So their numerical values can be more easily computed from the equivalent circuits than from the operational functions which are very complicated for more than two rotor circuits in each axis.

7. 7. NEGATIVE- AND ZERO-PHASE-SEQUENCE REACTANCE

As given in Chapter IV

$$X_z = \frac{X_d'' + X_q''}{2} \quad (7.84)$$

is the negative phase-sequence reactance for a sine-wave current and is defined as the negative phase-sequence reactance of a machine. The negative phase-sequence reactance for a sine-wave voltage is $\frac{2X_d''X_q''}{X_d'' + X_q''}$, while the negative phase-sequence reactance for a line-to-line short circuit is

$$\sqrt{X_d''X_q''}$$

The zero phase-sequence inductance is

$$L_o = \frac{6 \times 10^{-8}}{\pi} (k_p k_{an})^2 P_1 l (\lambda_{i0} + \lambda_{s0} + 0.2 \lambda_e) \quad (7.85)$$

The zero-phase-sequence reactance is

$$X_0 = \frac{L_0}{L_{e0}} = \frac{1}{3.19\pi} \frac{A g P_i}{F_g C_i R_a} (\lambda_{i0} + \lambda_{e0} + 0.2 \lambda_e) \quad (7.86)$$

where λ_{i0} , λ_{e0} and λ_e are specific permeances.

λ_e was given above, while λ_{i0} and λ_{e0} were derived by Kilgore and are as follows:-

$$\lambda_{i0} = \left[\lambda_i \left(\frac{kx_0}{k_x} \right) + \left(\frac{20}{39 k_p^2 k_d^2} \right) \left(\frac{h_i + 2 h_s}{12 b_s} \right) \right] \quad (7.87)$$

where λ_i was given in 7.4, and kx_0 is a coefficient for zero phase-sequence reactance and given in Fig. 7.4.

$$\lambda_{e0} = \left(\frac{kx_0}{k_x} \right) \frac{20 \tau_b}{\tau_r} \left[0.5 + \frac{d_b}{3 w_b} + \frac{d_r}{w_r} + \frac{8}{\tau_b} \right] \quad (7.88)$$

where τ_b = pitch of damper bars

d_b = depth of damper bar

w_b = width of damper bar

and the other dimensions were given above.

For a machine without damper bars,

$$\lambda_{e0} = \left(\frac{kx_0}{k_x} \right) 0.07 \times 3.19 \frac{4 R_a}{P_i g} \quad (7.89)$$

7.8. TIME CONSTANTS

As given by eq. (4.15), the open-circuit field (transient) time constant is

$$T_{do}' = \frac{X_{ffd}}{2\pi f R_{ffd}} \text{ in seconds or } \frac{X_{ffd}}{R_{ffd}} \text{ in radians} \quad (7.90)$$

where R_{ffd} is the field resistance at 75°C.

The unsaturated value of the short-circuit ^{field} (transient) ~~field~~ time constant is

$$T_{du}' = \left(\frac{X_{du}'}{X_d} \right) T_{do}' \quad (7.91)$$

which was shown in 4.10 where saturation was neglected.

Tests show that a good approximation to the saturated value may be obtained by assuming the field (transient) time constant to be reduced by the same factor as the transient reactance.

Therefore

$$T_d' = \frac{X_d'}{X_d} T_{do}' \quad (7.92)$$

The sub-transient time constants are much more difficult to calculate, especially when there are many additional rotor circuits. In reality each additional rotor circuit has its own time constant. But as the time constants of the additional rotor circuits in each axis do not differ much, it is well approximated to assume there is only one time constant for all the additional rotor circuits in each axis, which is termed the sub-transient time constant. As the field time constant, the sub-transient time constant has two values: the open-circuit and the short-circuit sub-transient time constant, according as the armature is open- or short-circuited.

Waring and Crary (40) showed that the short-circuit time constants can be found by impressing $-e_{do}1$ and $-e_{qo}1$ across the terminals and solving from the operational equations; and that the open-circuit time constants can be similarly found by impressing $-i_{do}1$ and $-i_{qo}1$. In most machines, the field or transient time constants are usually so many times as big as the sub-transient time constants that the transient time constant may be assumed to be infinity when calculating the sub-transient time constant and the sub-transient time constant to be zero in calculating the transient time constant without introducing much error. The armature resistance is also assumed to be zero. These assumptions greatly simplify the calculation.

For the case of one additional rotor circuit in each axis, Waring and Crary derived approximate formulae for the open-circuit and short-circuit sub-transient time constants.

$$T_{do}'' = \frac{X_{nd} - \frac{X_{fd}^2}{X_{td}}}{R_{nd}} \quad (7.93)$$

$$T_d'' = \frac{X_d''}{X_d'} \frac{X_{nd} - \frac{X_{fd}^2}{X_{td}}}{R_{nd}} \quad (7.94)$$

$$T_{q0}'' = \frac{X_{119}}{R_{119}} \quad (7.95)$$

$$T_q'' = \frac{x_q''}{x_q'} \frac{X_{119}}{R_{119}} \quad (7.96)$$

It is seen from the above equations that

$$T_d'' = \left(\frac{x_d''}{x_d'} \right) T_{d0}'' \quad (7.97)$$

$$T_q'' = \left(\frac{x_q''}{x_q'} \right) T_{q0}'' \quad (7.98)$$

The armature time constant was given in equation (4.20). In per-unit value (in radians) it is equal to

$$T_a = \frac{x_2}{r_a} \quad (7.99)$$

where r_a is the d.c. armature resistance at 75°C.

C H A P T E R V I I I

TEST METHODS FOR THE DETERMINATION OF MACHINE CONSTANTS

8. 1. UNSATURATED AND SATURATED VALUES

Since machine coefficients are affected by saturation and other practical considerations, they are not strictly "constants". But in practice it is sufficient and convenient to treat them as such. The values of direct-axis and quadrature-axis synchronous reactances are to be understood as the "unsaturated" values. The values of direct-axis transient and sub-transient reactances and short-circuit time constants are understood as those values effective for a dead three-phase short circuit, from rated voltage at no load, these being termed "saturated" values for convenience. The values of the negative phase-sequence reactance is understood as that value effective when negative phase-sequence fundamental current is circulated, the magnitude of the current being that of the negative phase-sequence current obtaining on a line-to-line short circuit, from rated voltage at no load, this being termed "saturated" value. Since saturation effects for zero phase-sequence currents in the range usually encountered are not large, the zero-phase-sequence reactance is taken as that value effective when zero-phase-sequence fundamental current equal to rated current is circulated, this being termed "rated-current" value or "unsaturated" value.

8. 2. SYNCHRONOUS REACTANCES

8. 2. (1) X_d from S.C.C. and airgap line

X_d can be determined from the S.C.C., which gives the sustained short-circuit current relation to field current. The field current corresponding to full-load armature current on the S.C.C., divided by the field current corresponding to normal voltage on the airgap line, is equal to X_d in its per unit value.

8. 2. (2) X_d and X_q from slip test

The machine is driven at a speed slightly different from the synchronous value and a balanced set of voltages is applied to the armature with the field circuit open. The armature current taken by the machine will undergo periodic changes in value at the frequency of the slip. The current will be at its minimum or its maximum according as the polar axis or the interpolar axis coincides with the coil axis. The ratio of the maximum phase voltage to the minimum armature phase current gives the X_d , and that of the minimum phase voltage to the maximum armature current the X_q .

8. 2. (3) X_q by large-motor method

The machine mechanically coupled in electrical quadrature to

a relatively very large synchronous motor and connected to the motor line is operated unexcited. The large motor is adjusted to nearly zero power factor, which is possible because of the relatively very small load. Then the large-motor armature current is nearly pure direct-axis component and the armature current in the tested machine is nearly pure quadrature-axis component because of its quadrature position with respect to the large motor. Since there is no excitation voltage in the tested machine, the ratio of the applied armature voltage to the corresponding armature current is the X_q .

8. 2. (4) X_d and X_q FROM STATIC REACTANCES BY LOCKED LINE-TO-LINE TEST

It was shown in Chapter IV that the subtransient reactances are equal to one half the line-to-line static reactances, i.e.,

$$\begin{aligned} X_d'' &= \frac{1}{2} X_{sd} \\ X_q'' &= \frac{1}{2} X_{sq} \end{aligned}$$

With no additional rotor circuits (or with the additional rotor circuits opened or removed) and with the field circuit open, and a line-to-line voltage applied to the machine, the ratio of the armature voltage to the armature current will vary cyclically from a maximum when the rotor is directly under the centre of the two phases considered as a single phase, to a minimum when the rotor is in quadrature thereto. Then half these ratios are the X_d and X_q respectively.

The value of X_d determined from (8.2.1.) is always the unsaturated value and those values of X_d and X_q determined from (8.2.2-4), if the applied phase voltages are relatively low, and the armature currents are of the magnitude of the order of rated current, are also unsaturated values, as required.

8. 3. ARMATURE LEAKAGE REACTANCE

X_l can be estimated by subtracting the calculated armature-reaction reactance from the test value of synchronous reactance.

$$\begin{aligned} X_l &= X_d \text{ (tested)} - X_{ad} \text{ (calculated)} \\ &= X_q \text{ (tested)} - X_{aq} \text{ (calculated)} \end{aligned}$$

It is possible to measure the total leakage reactance of a synchronous machine at standstill just as for an induction machine. It is also possible to measure it with the rotor removed and to determine the leakage reactance as the difference between the total and the calculated reactance due to the fundamental of the flux produced in the air core. It is also possible to insert exploding coils on the armature surface and by their means measure

the net flux existing during a short circuit test, which gives a measure of the leakage reactance.

All these methods are inaccurate. Comparatively the first method is the most straightforward, provided that adequate data are to hand. Clearly the calculated X_{ad} and X_{aq} must be reliable.

8. 4. TRANSIENT AND SUB-TRANSIENT REACTANCES

8. 4. (1) X_d' and X_d'' BY DEAD THREE-PHASE SHORT CIRCUIT TEST FROM RATED VOLTAGE AT NO LOAD

From the oscillogram of armature phase currents of a dead three-phase short circuit from rated voltage at no load can be found the asymmetrical component curves of the three phase-currents.

The factor for the asymmetrical component times the distance from the mid-point curve to the zero current line at any instant converts this distance to per unit current. It equals the amperes per unit **distance** of deflection divided by the rated r.m.s. value of current.

These curves are plotted for asymmetrical components on a semi-log paper. Since the asymmetrical component currents are exponential curves or nearly so, they appear as straight lines on the semi-log graph. By extrapolation to the zero time line their initial values can be obtained. These curves are roughly shown in Fig. 8.1.

The distances oa , ob , and oc in the above figure are the logarithms of the initial values of the asymmetrical components of the three phase currents. The maximum asymmetrical component om can be found by laying oa , ob , and oc at 60° with each other and drawing perpendiculars am , bm , and cm from the three points a , b , and c respectively. Theoretically the three perpendiculars should intersect at one point m . But in practice they rarely do, and usually there is a small triangular residual, the centre of which is taken as the point m . See Fig. 8.2.

Then the sub-transient current

$$i'' = \frac{om}{\sqrt{2}}$$

and

$$X_d'' = \frac{1.00}{i''}$$

Next the symmetrical component curve is plotted. The factor for the symmetrical component times the distance between envelopes converts this distance to per unit r.m.s. current. It equals $\frac{1}{\sqrt{2}}$ or 0.707 times the asymmetrical component factor.

The symmetrical curve is shown in Fig. 8.3.

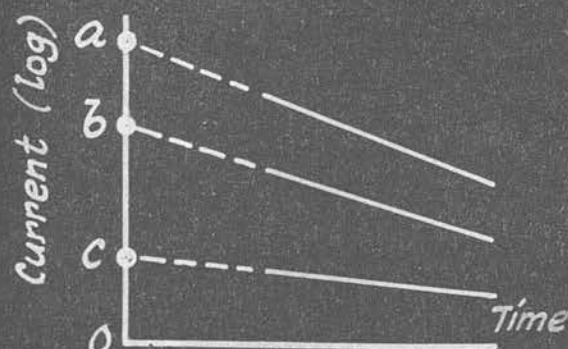


FIG. 8.1. Log. of asymmetric component currents to time base.

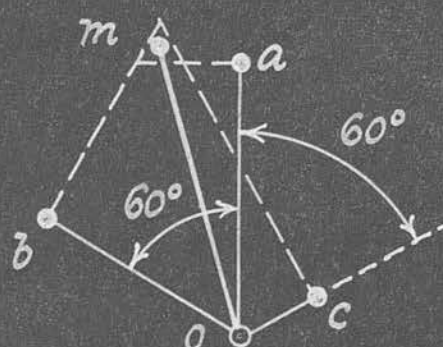


FIG. 8.2. Graphical method of obtaining O_m .

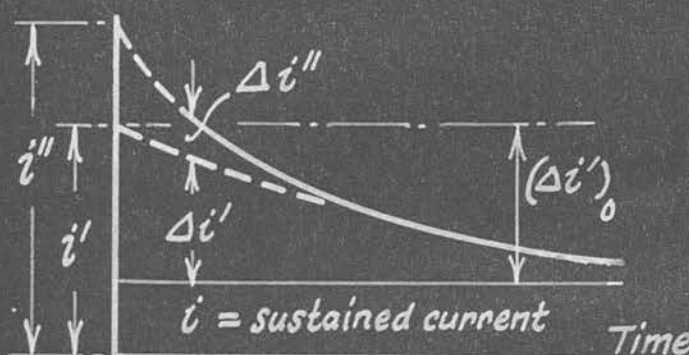


FIG. 8.3. Components of symmetrical armature phase current.

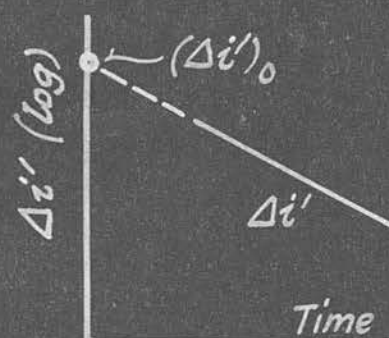


FIG. 8.4. Initial value of $\Delta i'$.

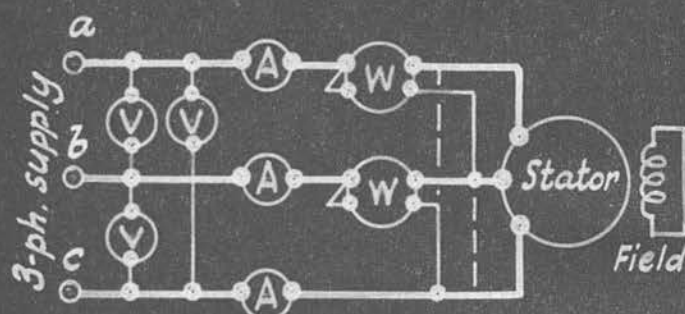


FIG. 8.5. Connections for Test I.

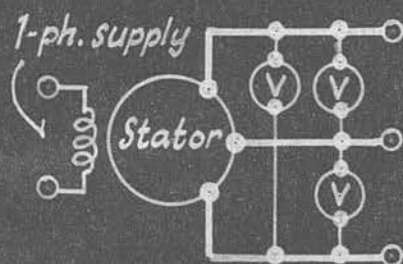


FIG. 8.6. Connections for Test II.

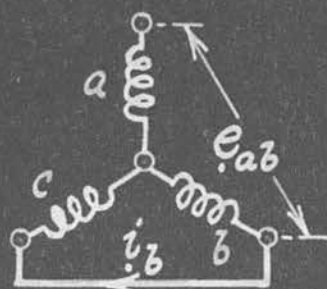


FIG. 8.7. Line-to-line sh. circuit.

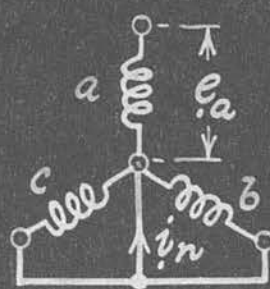


FIG. 8.8. Double line-to-neutral sh. circuit.

By plotting $\Delta i'$, the difference of the transient current and the sustained current i , on a semi-log paper, the initial value $(\Delta i')_0$ can be obtained, Fig. 8.4.

$$\text{Then } X_d' = \frac{1.00}{i + (\Delta i')_0} = \frac{1.00}{i'}$$

X_d'' and X_d' obtained from this test are the saturated values as required.

8. 4. (2) X_d' , X_q' , X_d'' , and X_q'' FROM STATIC REACTANCES BY LOCKED LINE-TO-LINE TEST

If there are no additional rotor circuits, or the additional rotor circuits can be open or removed, and with the field short-circuited, then half the static reactances measured are the transient reactances. Since in a salient-pole machine there is no field winding in the quadrature axis, the quadrature axis transient reactance is equal to the quadrature-axis synchronous reactance.

If all the additional rotor circuits as well as the field circuit are short-circuited, then half the measured static reactances will be the sub-transient reactances.

Since in this test the current is usually of the order of rated current, the reactances obtained are the unsaturated values. To change unsaturated values of transient and sub-transient reactances into saturated values, we either multiply them by a factor equal to the ratio of field amperes at rated voltage on the airgap line to field amperes at rated voltage on o.c.c., which ratio is simply the reciprocal of the product of X_d and the short-circuit-ratio; or by the empirical factor 0.88

8. 4. (3) X_d' FROM TIME-CONSTANTS

X_d' can be found from the time constants of the decay of field voltage and current respectively with the armature open-circuited and short circuited, the time constants being determined oscillographically.

$$\therefore T_d' = \frac{X_d'}{X_d} T_{d0}'$$

$$\therefore X_d' = X_d \frac{T_d'}{T_{d0}'} = 0.88 \frac{T_{du}'}{T_{d0}'} X_d$$

8. 4. (4) X_d'' and X_q'' BY TRACY AND TICE'S METHOD

Tracy and Tice (51) proposed a method of measuring the sub-transient impedances of a synchronous machine by impressing three-phase voltages on the machine at standstill. This method has the advantages that it does not involve the taking of oscillograms and the consequent uncertainty in their analyses, nor does it require that the rotor of the machine under test be set in certain

definite positions. Moreover, as three-phase, rather than single-phase voltages are impressed, the measurements are made under conditions more nearly simulating those of normal operation.

This method consists of making two tests on the machine at standstill with the rotor in any position whatsoever.

Test I. The connection diagram for this test is shown in Fig. 8.5. With the field structure blocked in any position whatsoever and the field short-circuited, three-phase voltages are applied to the stator terminals, these voltages being of such a magnitude that approximately rated current flows in the stator. Measurements are made of the three line-to-line voltages e_{ab} , e_{bc} , and e_{ca} , the three line currents i_a , i_b , and i_c , and four single-phase watt meter readings $W_{a(ab)}$, $W_{a(ac)}$, $W_{b(bc)}$, and $W_{b(ba)}$, where the first subscript in each case shows the line in which the current coil is connected, and the following pair of subscripts shows the lines between which the potential coil is connected.

Test II. With the rotor in the same position as in Test I, a single-phase alternating voltage of rated frequency and of approximately the same magnitude as the rated d.c. field voltage is impressed on the field terminals (see Fig. 8.6). The open-circuit terminal voltages e_{ab} , e_{bc} , and e_{ca} are read.

From Test I, by the method of symmetrical components, e_1 , e_2 , i_1 , and i_2 can be obtained,

where e_1 = the positive phase-sequence component of the voltage from line to neutral in phase a ,

e_2 = the negative phase-sequence component of the voltage from line to neutral in phase a ,

i_1 = the positive phase-sequence component of the current in line a , and

i_2 = the negative phase-sequence component of the current in line a .

From Test II and a set of curves given in Tracy and Tice's paper, the angle θ by which the direct axis leads the axis of phase a can be obtained.

Then as shown by Tracy and Tice, the sub-transient impedances are

$$Z_d'' = \frac{e_1 + e_2 \angle 2\theta}{i_1 + i_2 \angle 2\theta}$$

$$Z_q'' = \frac{e_1 - e_2 \angle 2\theta}{i_1 - i_2 \angle 2\theta}$$

If the resistance is small and neglected, the sub-transient

reactances are then equal to the sub-transient impedances.

$$\therefore X_d'' = Z_d''$$

$$X_q'' = Z_q''$$

8. 5. NEGATIVE PHASE-SEQUENCE REACTANCE

8. 5. (1) FROM SUB-TRANSIENT REACTANCES

As shown in Chapter IV

$$X_2 = \frac{X_d'' + X_q''}{2}$$

8. 5. (2) FROM SUSTAINED LINE-TO-LINE SHORT CIRCUIT TEST (AIEE METHOD)

In a line-to-line short circuit, the current i_b and the voltage across the open phase E_{ab} are known to be

$$i_b = -j \frac{\sqrt{3} E}{Z_1 + Z_2}$$

$$E_{ab} = \frac{3 E Z_2}{Z_1 + Z_2}$$

where E = generated voltage of phase a . See Fig. 8.5.

$$\therefore \frac{E_{ab}}{i_b} = \frac{\sqrt{3} Z_2}{-j}$$

$$\text{Or } Z_2 = \frac{-j}{\sqrt{3}} \frac{E_{ab}}{i_b}$$

If P_2 = power measured using E_{ab} and i_b ,

$$X_2 = \frac{E_{ab}}{\sqrt{3} i_b} \cdot \frac{P_2}{E_{ab} i_b}$$

If $P_2 \div E_{ab} i_b$, which means a negligible resistance component,

$$X_2 = \frac{E_{ab}}{\sqrt{3} i_b}$$

As shown by Thomas, the measured value of X_2 by this method is $(X_d'' + X_q'')/2$, not the negative phase-sequence reactance at the operating condition, $\sqrt{X_d'' X_q''}$, because the measured values of E_{ab} and i_b are the effective values instead of the fundamentals.

8. 6. ZERO-PHASE-SEQUENCE REACTANCE

8. 6. (1) FROM LOCKED ZERO-PHASE-SEQUENCE REACTANCE TEST

As X_0 is little affected by whether the rotor is running

or at standstill, it can be obtained from locked zero-phase-sequence reactance tests. With the field winding closed on itself and the rotor at standstill, single-phase voltage is applied to the three-phase. The ratio of voltage to current is χ_0 .

8. 6. (2) FROM SUSTAINED DOUBLE LINE-TO-NEUTRAL SHORT CIRCUIT TEST

With a double line-to-neutral short circuit on phases b and c at the machine terminals, the voltage on the open phase is:-

$$e_a = \frac{3 Z_0 Z_2 e}{Z_0 Z_1 + Z_0 Z_2 + Z_1 Z_2}$$

and the neutral current is:-

$$i_n = 3 i_0 = - \frac{3 e Z_2}{Z_0 Z_1 + Z_0 Z_2 + Z_1 Z_2}$$

$$\text{Thus } Z_0 = - \frac{e_a}{i_n}$$

$$\text{and } \chi_0 = \frac{e_a}{i_n}$$

8. 7. TIME CONSTANTS

8. 7. (1) ARMATURE TIME CONSTANT

T_a is taken as the time in seconds for the asymmetrical current in that phase of the two largest components which has initially the most gradual slope to decrease to 0.368 of its initial value.

T_a can also be obtained by circulating d.c. in two (or three) phases by applying d.c. voltage between two terminals (or between two terminals tied together and the third). The rotor is driven at rated speed. Then when the terminals are short-circuited, T_a is the time for the current to decay to 0.368 of its initial value.

T_a can be obtained from the armature resistance and the negative phase-sequence reactance.

$$T_a = \frac{\chi_2}{2 \pi f r_a}$$

where r_a is the d.c. phase resistance at 75°C.

8. 7. (2) DIRECT-AXIS ROTOR CIRCUITS TIME CONSTANTS

T_{do}' can be determined from an open-circuit field decrement test. With the machine running at about one-half rated voltage at no load, the field winding is suddenly short-circuited on itself and the armature voltage and field current recorded by oscillograph. T_{do}' is then obtained from the semi-log plot of armature voltage.

T_{du}' can be determined from a short-circuit field decrement

test. With a sustained three-phase short-circuit at the machine terminals and a fraction (about half) of rated current flowing, the field winding is suddenly short-circuited on itself, and an oscillographic record made of armature and field currents, the latter being recorded because it normally has a sudden initial change which helps in fixing zero time. A semi-log plot of armature current then gives T_{du}' .

T_d' is approximately equal to $0.88 T_{du}'$.

T_d' may also be determined from the semi-log plot of the $\Delta i'$ component of armature current of a dead three-phase short circuit from rated voltage at no load.

T_d' may also be determined from T_{do}' and transient reactance X_d' .

$$T_d' = \frac{X_d'}{X_d} T_{do}'$$

T_{do}'' can be obtained from a modification of the slip test with the field winding closed on itself; the rotor is magnetized in the d-axis, the applied voltage suddenly disconnected, and the terminal voltage recorded by oscillograph. From a semi-log plot of voltage, the slowly decaying component is determined; the difference between the total voltage and the slowly decaying component is plotted on semi-log paper and the time constant of the fitted exponential is T_{do}'' .

T_d'' can be determined from the semi-log plot of $\Delta i''$ component of armature current of a dead three-phase short circuit from rated voltage at no load.

T_d'' can also be obtained from T_{do}'' and sub-transient reactance.

$$T_d'' = \frac{X_d''}{X_d'} T_{do}''$$

8. 7. (3) QUADRATURE-AXIS ROTOR CIRCUITS TIME-CONSTANTS

As there is no field winding in the quadrature axis, there are no values representing T_{qo}' , T_{qu}' and T_q' .

T_{qo}'' can be determined from the large rotor method for the determination of X_q . The applied voltage is suddenly disconnected and the terminal voltage recorded by oscillograph. The semi-log plot of the terminal voltage gives T_{qo}'' .

$$T_q'' = \frac{X_q''}{X_q} T_{qo}''$$

C H A P T E R IX

MACHINE CHARACTERISTICS AND PERFORMANCE

9. 1. GENERAL

In the foregoing the "constants" of a salient-pole synchronous machine have been discussed in detail and the methods of their determination either by calculation or by test have been given. The machine characteristics and performance depend in general upon these constants and can be expressed in terms of them. In this chapter equations for the most important machine performance are furnished.

While many authors have studied aspects of performance and derived formulae for their expression mostly from the physical point of view, Park studied the machine by means of Heaviside's operational calculus and analyzed the machine performance on a basis of the fundamental equations derived by the mathematical method from flux linkage equations. While Park's analysis is rigid and exact and can be applied to any kind of machine performance, its derivation is rather complicated and cumbersome. Park's general method and fundamental equations are given in the following, and for the most important machine performance the derivation of formulae by other authors from the physical point of view is also reproduced.

9. 2. PARK'S OPERATIONAL METHOD OF ANALYSIS

The flux linkages formulae for an ideal synchronous machine were derived by Park and are as follows:-

$$\begin{aligned} \psi_a = & X_{ad} I_d \cos \theta - X_{aq} I_q \sin \theta - \frac{X_o}{3} (i_a + i_b + i_c) \\ & - \frac{X_d + X_q}{3} \left(i_a - \frac{i_b + i_c}{2} \right) \\ & - \frac{X_d - X_q}{3} \left[i_a \cos 2\theta + i_b \cos \left(2\theta - \frac{2\pi}{3} \right) + i_c \cos \left(2\theta + \frac{2\pi}{3} \right) \right] \end{aligned} \quad (9.1)$$

$$\begin{aligned} \psi_b = & X_{ad} I_d \cos \left(\theta - \frac{2\pi}{3} \right) - X_{aq} I_q \sin \left(\theta - \frac{2\pi}{3} \right) - \frac{X_o}{3} (i_b + i_c + i_a) \\ & - \frac{X_d + X_q}{3} \left(i_b - \frac{i_c + i_a}{2} \right) \\ & - \frac{X_d - X_q}{3} \left[i_b \cos \left(2\theta + \frac{2\pi}{3} \right) + i_c \cos 2\theta + i_a \cos \left(2\theta - \frac{2\pi}{3} \right) \right] \end{aligned} \quad (9.2)$$

$$\begin{aligned}\psi_c = & X_{ad} I_d \cos(\theta + \frac{2\pi}{3}) - X_{aq} I_q \sin(\theta + \frac{2\pi}{3}) - \frac{X_0}{3} (i_c + i_a + i_b) \\ & - \frac{X_d + X_q}{3} (i_c - \frac{i_a + i_b}{2}) \\ & - \frac{X_d - X_q}{3} [i_c \cos(2\theta - \frac{2\pi}{3}) + i_a \cos(2\theta + \frac{2\pi}{3}) + i_b \cos 2\theta]\end{aligned}\quad (9.3)$$

where $X_{ad} I_d = X_{afd} I_{fd} + X_{ard} I_{rd} + X_{zrd} I_{zd} + \dots$

$$X_{aq} I_q = X_{aq1} I_{q1} + X_{aq2} I_{q2} + \dots$$

The rotor currents I_{fd} , I_{rd} , I_{zd} , I_{q1} , I_{q2} , etc., can be found from the rotor flux linkages and voltage equations in terms of E_{fd} and i_d with operational coefficients, so that

$$X_{ad} I_d = G(p) E_{fd} + [X_d - X_d(p)] i_d \quad (9.4)$$

$$X_{aq} I_q = [X_q - X_q(p)] i_q \quad (9.5)$$

where $G(p)$, $X_d(p)$, and $X_q(p)$ are operational functions. For transient conditions we put $p = \alpha$ and for steady state conditions we put $p = 0$.

Now

$$i_d = \frac{2}{3} [i_a \cos \theta + i_b \cos(\theta - \frac{2\pi}{3}) + i_c \cos(\theta + \frac{2\pi}{3})] \quad (9.6)$$

$$i_q = -\frac{2}{3} [i_a \sin \theta + i_b \sin(\theta - \frac{2\pi}{3}) + i_c \sin(\theta + \frac{2\pi}{3})] \quad (9.7)$$

Let t = time in electrical radian

$$\text{Then } e_a = p\psi_a - r_a i_a \quad (9.8)$$

$$e_b = p\psi_b - r_a i_b \quad (9.9)$$

$$e_c = p\psi_c - r_a i_c \quad (9.10)$$

From equation (9.1) to (9.10) there are ten equations for 15 variables, viz. e_a , e_b , e_c , ψ_a , ψ_b , ψ_c , i_a , i_b , i_c , i_d , i_q , I_d , I_q , E_{fd} , and θ .

Therefore if five of the fifteen variables are known, the others can be found by solving the ten equations.

Now let us introduce i_0 , e_0 , ψ_0 , e_d , e_q , ψ_d , and ψ_q , where

$$i_0 = \frac{1}{3} (i_a + i_b + i_c)$$

$$e_0 = \frac{1}{3} (e_a + e_b + e_c)$$

$$\psi_0 = \frac{1}{3}(\psi_a + \psi_b + \psi_c)$$

$$e_{td} = \frac{2}{3} \left[e_a \cos \theta + e_b \cos \left(\theta - \frac{2\pi}{3} \right) + e_c \cos \left(\theta + \frac{2\pi}{3} \right) \right]$$

$$e_{tq} = -\frac{2}{3} \left[e_a \sin \theta + e_b \sin \left(\theta - \frac{2\pi}{3} \right) + e_c \sin \left(\theta + \frac{2\pi}{3} \right) \right]$$

$$\psi_d = \frac{2}{3} \left[\psi_a \cos \theta + \psi_b \cos \left(\theta - \frac{2\pi}{3} \right) + \psi_c \cos \left(\theta + \frac{2\pi}{3} \right) \right]$$

$$\psi_q = -\frac{2}{3} \left[\psi_a \sin \theta + \psi_b \sin \left(\theta - \frac{2\pi}{3} \right) + \psi_c \sin \left(\theta + \frac{2\pi}{3} \right) \right]$$

By substituting these quantities into the ten equations above, we obtain the following six equations.

$$e_{td} = p\psi_d - r_a i_d - \psi_q p\theta \quad (9.11)$$

$$e_{tq} = p\psi_q - r_a i_q + \psi_d p\theta \quad (9.12)$$

$$e_0 = p\psi_0 - r_a i_0 \quad (9.13)$$

$$\psi_d = \chi_{ad} I_d - \chi_d i_d = G(p) E_{td} - \chi_d(p) i_d \quad (9.14)$$

$$\psi_q = \chi_{aq} I_q - \chi_q i_q = -\chi_q(p) i_q \quad (9.15)$$

$$\psi_0 = -\chi_0 i_0 \quad (9.16)$$

Then the ten equations for fifteen quantities are reduced to six equations for eleven quantities e_{td} , e_{tq} , e_0 , i_d , i_q , i_0 , ψ_d , ψ_q , E_{td} , and θ .

Knowing these quantities, i_a , i_b , i_c , e_a , e_b , e_c , ψ_a , ψ_b , and ψ_c can be found from the following relations.

$$i_a = i_d \cos \theta - i_q \sin \theta + i_0$$

$$i_b = i_d \cos \left(\theta - \frac{2\pi}{3} \right) - i_q \sin \left(\theta - \frac{2\pi}{3} \right) + i_0$$

$$i_c = i_d \cos \left(\theta + \frac{2\pi}{3} \right) - i_q \sin \left(\theta + \frac{2\pi}{3} \right) + i_0$$

$$e_a = e_{td} \cos \theta - e_{tq} \sin \theta + e_0$$

$$e_b = e_{td} \cos \left(\theta - \frac{2\pi}{3} \right) - e_{tq} \sin \left(\theta - \frac{2\pi}{3} \right) + e_0$$

$$e_c = e_{td} \cos \left(\theta + \frac{2\pi}{3} \right) - e_{tq} \sin \left(\theta + \frac{2\pi}{3} \right) + e_0$$

$$\psi_a = \psi_d \cos \theta - \psi_q \sin \theta + \psi_0$$

$$\psi_b = \psi_d \cos \left(\theta - \frac{2\pi}{3} \right) - \psi_q \sin \left(\theta - \frac{2\pi}{3} \right) + \psi_0$$

$$\psi_c = \psi_d \cos \left(\theta + \frac{2\pi}{3} \right) - \psi_q \sin \left(\theta + \frac{2\pi}{3} \right) + \psi_0$$

Let $e_t = e_{td} + j e_{tq}$

$$i = i_d + j i_q$$

$$\psi = \psi_d + j \psi_q$$

$$\text{Then } \chi_a \dot{I} = \chi_{ad} \dot{I}_d + j \chi_{aq} \dot{I}_q \quad (9.17)$$

$$E_t = p\psi - r_a i + [p\psi] j \quad (9.18)$$

$$\psi = \chi_a \dot{I} - \chi \dot{i}$$

$$\text{Where } \chi \dot{i} = \chi_d i_d + j \chi_q i_q$$

These vectors can be expressed in a vector diagram as Fig. 9.1

9.3. APPLICATION OF PARK'S FUNDAMENTAL EQUATIONS

The fundamental equations in 9.2 can be applied to the analysis of any machine performance, and Park himself gave some of these applications in his papers on two-reaction theory. Some of the equations derived in his papers are quoted:-

9.3. (1) ARMATURE OUTPUT

$$\begin{aligned} P &= \text{per-unit instantaneous power output} \\ &= \frac{2}{3} (e_a i_a + e_b i_b + e_c i_c) \\ &= e_{ad} i_d + e_{aq} i_q + e_o i_o \end{aligned} \quad (9.19)$$

9.3. (2) ELECTRICAL TORQUE ON ROTOR

$$\begin{aligned} \text{Since } P &= T p\psi - \frac{2}{3} r_a (i_a^2 + i_b^2 + i_c^2) \\ &= T p\psi - r_a (i_d^2 + i_q^2 + i_o^2) \\ \text{Then } T &= \frac{e_{ad} i_d + e_{aq} i_q + e_o i_o + r_a (i_d^2 + i_q^2 + i_o^2)}{p\psi} \end{aligned}$$

Assume the simplest condition of operation, i.e., steady state operation at synchronous speed, then I_d , I_q , i_d , i_q , ψ_d , ψ_q , ψ_o , and i_o are all constant.

$$\begin{aligned} \therefore e_{ad} &= -\psi_q p\psi - r_a i_d \\ e_{aq} &= \psi_d p\psi - r_a i_q \\ e_o &= -r_a i_o \\ \therefore T &= i_q \psi_d - i_d \psi_q \\ &= \text{vector product of } \psi \text{ and } i \\ &= \psi \times i \end{aligned} \quad (9.20)$$

9.3. (3) CONSTANT ROTOR SPEED WITH CONSTANT SLIP

Let slip = S

$$\begin{aligned} e_{ad} &= p\psi_d - r_a i_d - (1-S) \psi_q \\ e_{aq} &= p\psi_q - r_a i_q + (1-S) \psi_d \end{aligned}$$

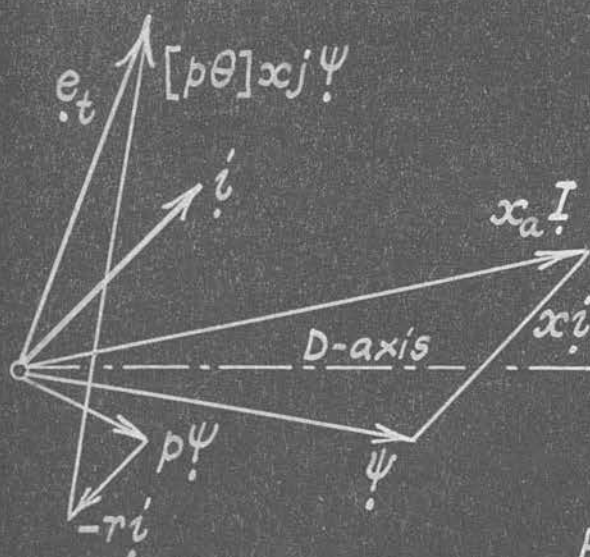


FIG. 9.1. Park's vector diagram.

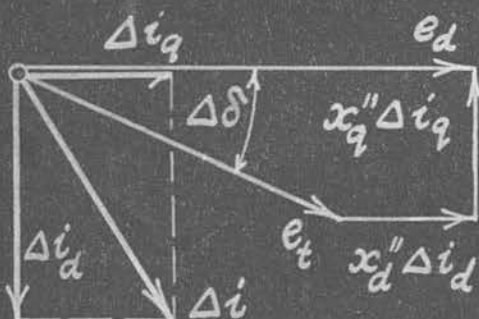


FIG. 9.5. Vector diagram for synchronizing out of phase.

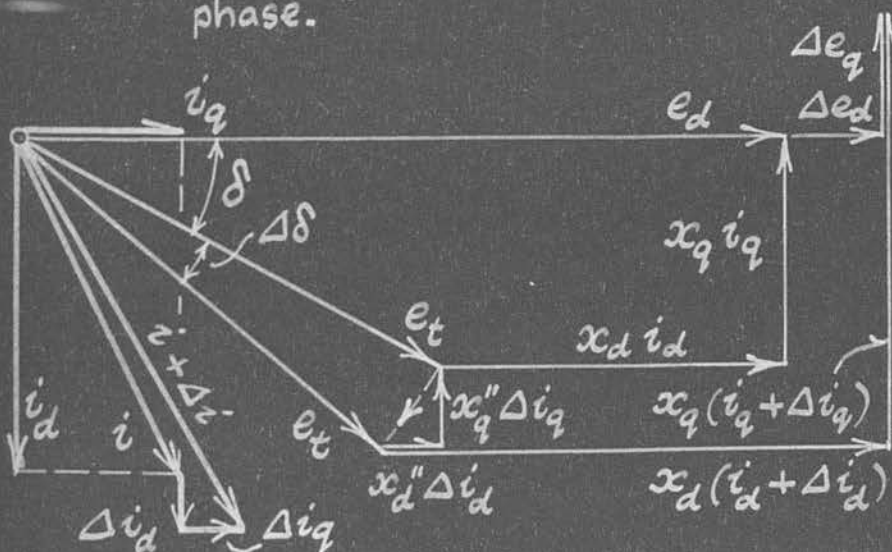


FIG. 9.4. Vector diagram of salient-pole machine under sudden angular displacement.

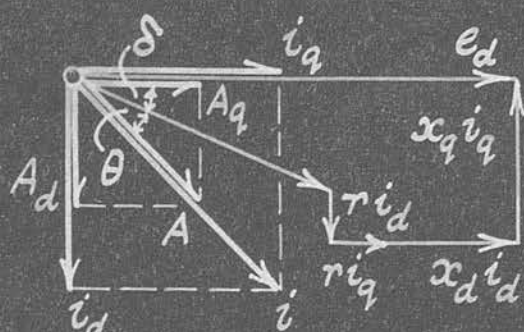


FIG. 9.2. Steady-state vector diagram of salient-pole machine.

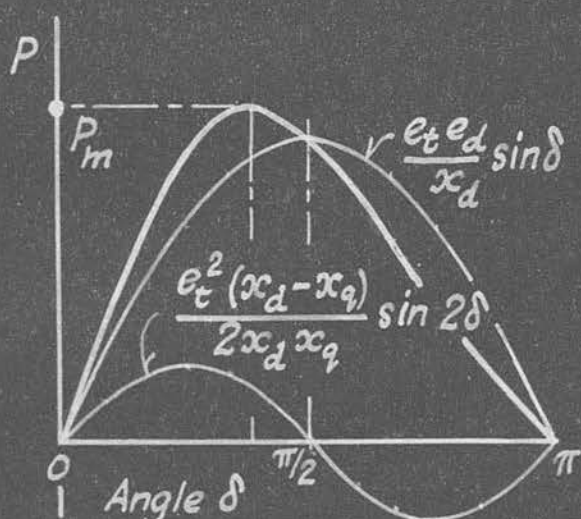


FIG. 9.3. Power/angle curve.

But $\psi_d = G(p) E_{fd} - X_d(p) i_d$

$\psi_q = -X_q(p) i_q$

Putting $p X_d(p) + r_a = Z_d(p)$

$p X_q(p) + r_a = Z_q(p)$

$\therefore E_{td} = p G(p) E_{fd} - Z_d(p) i_d + (1-s) X_q(p) i_q$

$E_{tq} = (1-s) [G(p) E_{fd} - X_d(p) i_d] - Z_q(p) i_q$

Solving for i_d and i_q ,

$$i_d = \left\{ [p Z_q(p) + (1-s)^2 X_q(p)] G(p) E_{fd} - Z_q(p) E_{td} - (1-s) X_q(p) E_{tq} \right\} \div D(p) \quad (9.21)$$

$$i_q = \left\{ (1-s) r_a G(p) E_{fd} - Z_d(p) E_{tq} + (1-s) X_d(p) E_{td} \right\} \div D(p) \quad (9.22)$$

Where $D(p) = Z_d(p) Z_q(p) + (1-s)^2 X_d(p) X_q(p)$

9.3. (4) TORQUE-ANGLE RELATIONS

$$\begin{aligned} T &= i_q \psi_d - i_d \psi_q \\ &= \psi_d \left(\frac{X_{aq} I_q - \psi_q}{X_q} \right) - \psi_q \left(\frac{X_{ad} I_d - \psi_d}{X_d} \right) \\ &= \frac{X_{aq} I_q \psi_d}{X_q} - \frac{X_{ad} I_d \psi_q}{X_d} - \frac{X_d - X_q}{X_d X_q} \psi_d \psi_q \end{aligned} \quad (9.23)$$

If the rotor leads the vector ψ by an angle δ , there is

$\psi_d = \psi \cos \delta$

$\psi_q = -\psi \sin \delta$

$$\therefore T = \frac{X_{aq} I_q \psi}{X_q} \cos \delta + \frac{X_{ad} I_d \psi}{X_d} \sin \delta + \frac{X_d - X_q}{2 X_d X_q} \psi^2 \sin 2\delta \quad (9.24)$$

9. 3. (5) FURTHER ANALYSES

Park also derived equations for (a) two machines connected together, (b) one machine on an infinite bus-bar system, (c) three-phase short circuit with constant rotor speed, (d) starting torque, (e) zero armature resistance, one machine connected to an infinite bus-bar, (f) torque-angle- relation of a synchronous machine connected to an infinite bus, for small angular deviation for an average operating angle, (g) synchronizing and damping torque for small oscillations, (h) damping torque during disturbance etc.

Park's two-reaction theory has been extended by several authors to more general and complicated conditions. For instance, Ku extended Park's two reaction theory to multi-phase synchronous machines. Cray extended it to the case of machines having balanced three-phase capacitance in the armature circuit: and Concordia applied the theory to synchronous machines with any balanced terminal impedance. As the derivations are mostly mathematical and are very cumbersome, they are not reproduced here.

9. 4. STEADY-STATE POWER-ANGLE CHARACTERISTICS

The steady-state power-angle characteristics of a salient-pole machine can easily be seen from the vector diagram, which is given in Fig. 9.2.

E_d is the nominal or field excitation voltage in the direct-axis and E_t is the terminal voltage. The other vectors have the same significance as in the previous chapters.

The steady-state power is, from the vector diagram, equal to

$$P = E_t \cos \delta i_q + E_t \sin \delta i_d$$

$$\text{Now } X_q i_q - r_a i_d = E_t \sin \delta$$

$$E_d - X_d i_d - r_a i_q = E_t \cos \delta$$

Solving for i_d and i_q ,

$$i_d = \frac{X_q E_d - E_t (r_a \sin \delta + X_q \cos \delta)}{r_a^2 + X_d X_q}$$

$$i_q = \frac{E_d r_a - E_t (r_a \cos \delta - X_d \sin \delta)}{r_a^2 + X_d X_q}$$

Substituting into the power equation ,

$$P = \frac{E_t^2}{r_a^2 + X_d X_q} \left\{ \frac{E_d}{E_t} (r_a \cos \delta + X_q \sin \delta) + (X_d - X_q) \cos \delta \sin \delta - r_a \right\} \quad (9.25)$$

If r_a is very small in comparison with X_d and X_q , it may be neglected. Then equation (9.25) reduced to

$$\begin{aligned} P &= \frac{E_t E_d}{X_d} \sin \delta + \frac{E_t^2 (X_d - X_q)}{X_d X_q} \sin \delta \cos \delta \\ &= \frac{E_t E_d}{X_d} \sin \delta + \frac{E_t^2 (X_d - X_q)}{2 X_d X_q} \sin 2\delta \end{aligned} \quad (9.26)$$

Equation (9.26) is the same as Park's torque equation (9.24) given above. Since r_a is neglected, E_t is equivalent to ψ . E_d is equivalent to $X_{ad} I_d$. As there is no excitation in the quadrature axis and a steady-state exists, $I_q = 0$. Thus these two equations are identical.

In a cylindrical-rotor machine, where $X_d = X_q$, the second term in equation (9.26) vanishes and the power is therefore proportional to $\sin \delta$. But in a salient-pole machine where there is difference between X_d and X_q , a second term appears, proportional to $\sin 2\delta$, as a second harmonic of the power angle. The excitation voltage E_d does not appear in the second term, which is consequently independent of the excitation. Even if there is no excitation power can still be developed. This power is entirely due to the difference of reluctance in the two axes, and is consequently termed the "reluctance power". Langsdorf analyzed the steady-state characteristics of a salient-pole machine and found that the current locus of a salient-pole machine for constant excitation is, unlike that of a cylindrical-rotor machine, not exactly a circle, but of a shape distorted therefrom. When the excitation is below a critical value, a re-entrant loop appears in the current locus. With zero excitation, the doubly-re-entrant loops degenerate to two complete traverses of a circle, half of which is within the circle of zero power locus. This again shows that power can be developed at zero excitation. In the case of a cylindrical-rotor machine the current locus for zero excitation is only a point on the circle of zero power.

The power/angle curve for a salient-pole machine is shown in Fig. 9.3.

In a cylindrical-rotor machine, if r_a is neglected, the maximum power occurs at $\delta = 90^\circ$. But in a salient-pole machine, as

seen from Fig. 8.3, the maximum power occurs at an angle δ_m smaller than 90° . It is also shown in the figure that for the same power output the power-angle of a salient-pole machine is much smaller than that of a comparable cylindrical-rotor machine of the same rating.

The synchronizing power is:-

$$P_s = \frac{dP}{d\delta} = \frac{E_t E_d}{X_d} \cos \delta + \frac{E_t^2 (X_d - X_q)}{X_d X_q} \cos 2\delta \quad (9.27)$$

It also contains a second harmonic term which does not exist in the case of a cylindrical rotor machine. It is seen from equation (9.27) that when $\delta < 45^\circ$, the synchronizing power of a salient-pole machine is greater than that of a cylindrical-rotor machine of the same rating, and therefore the salient-pole machine has a "stiffer" coupling. But when $\delta > 45^\circ$, the salient-pole machine is less stiff.

9. 5. TRANSIENT POWER-ANGLE CHARACTERISTICS

9. 5. (1) SUDDEN ANGULAR DISPLACEMENT

Let $\Delta\delta$ = increment of power-angle after sudden angular displacement.

Δi_d = increment of direct-axis component of current

Δi_q = increment of quadrature-axis component of current

ΔE_d = increment of direct-axis component of excitation voltage

ΔE_q = increment of quadrature-axis component of excitation voltage

Then if the terminal voltage E_t is assumed to remain constant and the resistance r_a is neglected the vector diagram will be as in Fig. 9.4.

As seen from the diagram, the transient increments in excitation voltages, ΔE_d and ΔE_q , can be taken into account by using sub-transient reactances X_d'' and X_q'' for the increments in current components Δi_d and Δi_q to form the reactance drops $X_d'' \Delta i_d$ and $X_q'' \Delta i_q$. If instead the voltage increments ΔE_d and ΔE_q are given, then the synchronous reactances X_d and X_q must be used for the current increments too.

From the vector diagram,

$$i_d = \frac{E_d - E_t \cos \delta}{X_d}$$

$$i_q = \frac{E_t \sin \delta}{X_q}$$

$$\Delta i_d = \frac{E_t [\cos \delta - \cos (\delta + \Delta \delta)]}{X_d''}$$

$$\Delta i_q = \frac{E_t [\sin (\delta + \Delta \delta) - \sin \delta]}{X_q''}$$

The power after sudden angular displacement is equal to:-

$$P = E_t \cos (\delta + \Delta \delta) \cdot (i_q + \Delta i_q) + E_t \sin (\delta + \Delta \delta) \cdot (i_d + \Delta i_d)$$

Substituting the values of i_d , i_q , Δi_d , and Δi_q into the power equation,

$$\begin{aligned} P &= E_t \cos (\delta + \Delta \delta) \left\{ \frac{E_t \sin \delta}{X_q} + \frac{E_t [\sin (\delta + \Delta \delta) - \sin \delta]}{X_q''} \right\} \\ &\quad + E_t \sin (\delta + \Delta \delta) \left\{ \frac{E_d - E_t \cos \delta}{X_d} + \frac{E_t [\cos \delta - \cos (\delta + \Delta \delta)]}{X_d''} \right\} \\ &= \frac{E_t E_d}{X_d} \sin (\delta + \Delta \delta) + E_t^2 \frac{X_d'' - X_q''}{2 X_d'' X_q''} \sin 2 (\delta + \Delta \delta) \\ &\quad + E_t^2 \frac{X_d - X_d''}{X_d X_d''} \sin (\delta + \Delta \delta) - E_t^2 \frac{X_q - X_q''}{X_q X_q''} \cos (\delta + \Delta \delta) \sin \delta \end{aligned} \quad (9.28)$$

The synchronizing power is

$$\begin{aligned} P_s &= \frac{dP}{d(\delta + \Delta \delta)} = \frac{\delta P}{\delta \Delta \delta} \\ &= \frac{E_t E_d}{X_d} \cos (\delta + \Delta \delta) + E_t^2 \frac{X_d'' - X_q''}{X_d'' X_q''} \cos 2 (\delta + \Delta \delta) \\ &\quad + E_t^2 \frac{X_d - X_d''}{X_d X_d''} \cos (\delta + \Delta \delta) \cos \delta + E_t^2 \frac{X_q - X_q''}{X_q X_q''} \sin (\delta + \Delta \delta) \sin \delta \end{aligned} \quad (9.29)$$

$$\begin{aligned}
 P_s = & \frac{E_t E_d}{x_d} \cos \delta + E_t^2 \frac{x_d'' - x_q''}{x_d'' x_q''} \cos 2\delta \\
 & + E_t^2 \frac{x_d - x_d''}{x_d x_d''} \cos^2 \delta + E_t^2 \frac{x_q - x_q''}{x_q x_q''} \sin^2 \delta
 \end{aligned} \tag{9.30}$$

9. 5. (2) CYCLIC VARIATION OF IMPRESSED TORQUE

Doherty and Nickle analyzed the case of cyclic variation of impressed torque with one rotor circuit in each axis. As the rotor resistances affect the currents and therefore the power and torque, sub-transient reactances cannot be used in the vector diagram. It was shown by them that there is a time phase angle between the cyclic variations of the direct-axis and quadrature-axis components of current, if the cyclic variations of current components are assumed to be harmonics of time. If the rotor-circuit resistances are very small and can be neglected, this phase angle vanishes and the case can be represented by the same vector diagram as Fig. 9.4 in (9.5.1).

The electrical torque and synchronizing torque then can be found by solving the torque equation which contains all the terms of electrical torque, damping torque, inertia torque, and impressed torque. Doherty and Nickle solved the torque equation by replacing $\frac{d}{dt}$ with $j\omega$. But if the torque equation is a non-linear differential equation, it usually is very difficult to solve and an integrator or a differential analyzer may be needed.

The following results were obtained by Doherty and Nickle in their analysis of the cyclic variation of impressed torque.

- (i) The synchronizing torque T_s under oscillating condition is same as the steady-state synchronizing torque at $\delta = \delta_0$ (neglecting r_a) and departs only slightly up to $\delta = 30^\circ$.
- (ii) T_s is still positive beyond the maximum power point of stable steady-state operation.
- (iii) The difference between the steady-state synchronizing torque and the maximum slope of T_s is much greater for cylindrical-rotor machines than for salient-pole machines.
- (iv) The existence of quadrature-axis circuits causes the slope to be steeper, being positive between $\delta = 0$ and $\delta = \pi$.

9. 5. (3) SYNCHRONIZING OUT-OF-PHASE

Synchronizing out-of-phase is equivalent to the case of

(9.51), i.e., sudden angular displacement with i_d , i_q , and δ equal to zero and the initial terminal voltage equal to E_d . The machine is connected to a terminal voltage E_t at an angle $\Delta\delta$ to and there is suddenly a current Δi . The vector diagram is shown in Fig. 9.5.

This vector diagram is same as that of steady-state except that the synchronous reactances are replaced by sub-transient reactances.

From the diagram,

$$\Delta i_d = \frac{E_d - E_t \cos \Delta\delta}{X_d''}$$

$$\Delta i_q = \frac{E_t \sin \Delta\delta}{X_q''}$$

$$P = E_t \sin \Delta\delta \cdot \Delta i_d + E_t \cos \Delta\delta \cdot \Delta i_q$$

$$= \frac{E_d E_t}{X_d''} \sin \Delta\delta + \frac{E_t^2 (X_d'' - X_q'')}{2 X_d'' X_q''} \sin 2\Delta\delta \quad (9.31)$$

This equation is similar to the steady-state power equation. It is the average value existing in the first moment under the transient condition. There are in addition alternating components of normal and higher frequencies which are not taken into account here. While the latter are of large magnitude, nevertheless the frequency is so high that the torque does not have time except in case of resonance to produce much displacement and therefore strain in the shaft.

9. 6. SINGLE-PHASE SHORT CIRCUIT

The single-phase short circuit of a salient-pole machine has been analyzed by Doherty and Nickle by means of the constant linkage theorem based on the following assumptions.

- (A) Negligible saturation,
- (B) A sine-wave open-circuit voltage,
- (C) One additional rotor circuit in the quadrature-axis,
- (D) Resistances of armature and field circuits negligible in calculating the magnitude of the initial short-circuit current, and the armature resistance in calculating the magnitude of the sustained current.
- (E) The machine short-circuited at no load.

The variable component of armature inductance varies between

the d-axis value and the q-axis value as a second harmonic function of the electrical space angle, as was shown by Doherty and Nickle.

$$l' = \frac{1}{2} [(l_d' + l_q') + (l_d' - l_q') \cos 2\gamma]$$

where γ is the angle the direct axis leads the axis of the short-circuiting phase.

In per-unit values

$$l' = \frac{1}{2} [(x_d' + x_q') + (x_d' - x_q') \cos 2\gamma]$$

Let $\gamma = \tau + \alpha$

Where α is the angle by which the direct-axis leads the axis of the short-circuiting phase at the instant of short circuit.

Then $\psi_i = l' i = \frac{1}{2} i [(x_d' + x_q') + (x_d' - x_q') \cos 2(\tau + \alpha)]$

$$\psi_m = k e_{ao} \cos(\tau + \alpha)$$

$$\psi_m' = k e_{ao} \cos \alpha$$

where e_{ao} = peak value of phase voltage, and therefore also the flux linkage

ψ_i = flux linkage due to short-circuit current

ψ_m = flux linkage due to the field

ψ_m' = flux linkage due to the field just before short circuit

k = 1.0 for line-to-neutral, and 1.73 for line-to-line short circuit

By the constant-linkage theorem, the total flux linkage after short-circuit is equal to that before the short circuit. And since ψ_i is opposite to flux linkage due to field,

$$-\psi_i + \psi_m = \psi_m'$$

Or

$$-\frac{1}{2} i [(x_d' + x_q') + (x_d' - x_q') \cos 2(\tau + \alpha)] + k e_{ao} \cos(\tau + \alpha) = k e_{ao} \cos \alpha \quad (9.32)$$

While Doherty and Nickle assume only one rotor circuit in each axis, i.e., a field circuit but no additional rotor circuits in the direct axis, and one additional rotor circuit in the quadrature axis, the same principle method of analysis can be applied to

the case of any number of additional rotor circuits in each axis, and eq. (9.32) holds if the transient static reactances X_d' and X_q' are replaced by sub-transient static reactances X_d'' and X_q'' .

$$\begin{aligned} \therefore -\frac{1}{2} i [(X_d'' + X_q'') + (X_d'' - X_q'') \cos 2(t + \alpha)] + k E_{ao} \cos(t + \alpha) \\ = k E_{ao} \cos \alpha \end{aligned} \quad (9.33)$$

Now we can change the static reactances into three-phase reactances:-

For line-to-line short circuit,

$$X_D'' = X_{sd} = 2X_d''$$

$$X_Q'' = X_{sq} = 2X_q''$$

$$\text{And since } k = 1.73 = \sqrt{3}$$

$$X_D'' = \frac{2}{3} k^2 X_d''$$

$$X_Q'' = \frac{2}{3} k^2 X_q''$$

(9.34)

For single-phase line-to-neutral short circuit

$$X_D'' = X_{rd} = \frac{X_0 + 2X_d''}{3}$$

$$X_Q'' = X_{rq} = \frac{X_0 + 2X_q''}{3}$$

X_0 is usually small as compared with the sub-transient reactances, so that it may be neglected; and in the case of single-phase short circuit, $k = 1$,

$$\therefore X_D'' = \frac{2}{3} k^2 X_d''$$

$$X_Q'' = \frac{2}{3} k^2 X_q''$$

(9.34)

Then the same relation between the static reactances and three-phase reactances is obtained for either line-to-line or line-to-neutral short circuit except for the different values of k . This relation was derived by Doherty and Nickle by another method.

If X_0 is not negligible, X_d'' and X_q'' must be replaced by $X_d'' + X_0/2$ and $X_q'' + X_0/2$ respectively in the case of a line-to-neutral short circuit.

Substituting (9.34) into (9.33)

$$-\frac{1}{3} i k [(X_d'' + X_q'') + (X_d'' - X_q'') \cos 2(t + \alpha)] + E_{ao} \cos(t + \alpha)$$

$$= E_{ao} \cos \alpha$$

(9.35)

$$i = \frac{3 E_{a0} [\cos(t+\alpha) - \cos \alpha]}{k[(X_d'' + X_q'') + (X_d'' - X_q'') \cos 2(t+\alpha)]}$$

$$= \frac{3 E_{a0} [\cos(t+\alpha) - \cos \alpha]}{k H} \quad (9.36)$$

Where $H = (X_d'' + X_q'') + (X_d'' - X_q'') \cos 2(t+\alpha)$ ~~(9.36)~~
 Or

$$i = \frac{3 E_{a0}}{k(X_d'' + X_{s2})} 0 - \frac{3 E_{a0} \cos \alpha}{2k X_{s2}} E \quad (9.37)$$

where $X_{s2} = \sqrt{X_d'' X_q''}$ = the negative phase-sequence reactance for single-phase short circuit. ($X_2 = (X_d'' + X_q'')/2$ = negative phase-sequence reactance for sine-wave balanced phase-sequence current).

$$O = \text{odd series} = \frac{(X_d'' + X_{s2}) \cos(t+\alpha)}{H}$$

$$= \cos(t+\alpha) + b \cos 3(t+\alpha) + b^2 \cos 5(t+\alpha) + \dots$$

$$= \sum_{n=1,2,3,\dots} b^{\frac{n-1}{2}} \cos n(t+\alpha)$$

$$E = \text{even series} = \frac{2 X_{s2}}{H}$$

$$= 1 + 2b \cos 2(t+\alpha) + 2b^2 \cos 4(t+\alpha) + \dots$$

$$= 1 + \sum_{n=2,4,\dots} 2b^{\frac{n}{2}} \cos n(t+\alpha)$$

and $b = \frac{\sqrt{X_q''} - \sqrt{X_d''}}{\sqrt{X_q''} + \sqrt{X_d''}} = \frac{X_{s2} - X_d''}{X_{s2} + X_d''}$

The direct-axis component of the armature current is:-

$$i_d = \frac{2k}{3} i \cos(t+\alpha)$$

$$= \frac{2 E_{a0}}{X_d'' + X_{s2}} 0 \cos(t+\alpha) - \frac{E_{a0} \cos \alpha}{X_{s2}} E \cos(t+\alpha)$$

The average value of the even-series components of current is zero, and the average value of the odd-series components of current is $E_{a0} / (X_d'' + X_{s2})$.

The increment in d.c. excitation is:-

$$X_{ad} \Delta'' I_d (d.c.) = \frac{(X_d - X_d'') E_{ao}}{X_d'' + X_{s2}}$$

The total d.c. excitation is:-

$$E_{ao} + X_{ad} \Delta'' I_d (d.c.) = E_{ao} \frac{X_d + X_{s2}}{X_d'' + X_{s2}} \quad (9.38)$$

The average value of the direct-axis component of armature current due to the linkages supported by the field winding alone is:-

$$\frac{E_{ao}}{X_d' + X_{s2}}$$

The d.c. excitation due to the main field current alone is:-

$$E_{ao} + X_{ad} \Delta' I_d (d.c.) = E_{ao} \frac{X_d + X_{s2}}{X_d' + X_{s2}} \quad (9.39)$$

The difference between this excitation and the excitation E_{ao} , which is supported by the exciter, decays according to the field decrement T_{ds}' . Likewise the difference between (9.38) and (9.39) decays according to the direct-axis damper winding decrement T_{ds}'' . ~~There~~

Therefore the transient total d.c. excitation is:-

$$X_{ad} I_d (d.c.) = \frac{X_d + X_{s2}}{X_d'' + X_{s2}} F E_{ao} \quad (9.40)$$

where

$$F = \frac{X_d'' + X_{s2}}{X_d + X_{s2}} + \frac{(X_d'' + X_{s2})(X_d - X_d')}{(X_d + X_{s2})(X_d' + X_{s2})} e^{-\frac{T_{ds}'}{T_{ds}''} t} + \frac{X_d' - X_d''}{X_d' + X_{s2}} e^{-\frac{T_{ds}''}{T_{ds}'} t}$$

Since the odd-series components of armature current are due to rotor linkages the magnitudes of these components at any time after short circuit are F times their initial magnitudes.

Now $A = e^{-\frac{T_a}{T_a} t} \cos \alpha$ is chosen to represent the

trapped armature linkages at any time after short circuit.

$$\therefore i = \left(\frac{3}{2k}\right) \frac{2E_{ao} [F \cos(t + \alpha) - A]}{H} \quad (9.41)$$

Or

$$i = \frac{3k}{2k} \left[\frac{2E_{ao} F}{X_d'' + X_{s2}} 0 - \frac{E_{ao} A}{X_{s2}} E \right] \quad (9.42)$$

$$\begin{aligned} i_d &= \frac{2k}{3} i \cos(t + \alpha) \\ &= \frac{2E_{ao} [F \cos(t + \alpha) - A] \cos(t + \alpha)}{H} \\ &= \frac{2E_{ao} F}{X_d'' + X_{s2}} 0 \cos(t + \alpha) - \frac{E_{ao} A}{X_{s2}} E \cos(t + \alpha) \end{aligned} \quad (9.43)$$

$$\begin{aligned} i_q &= -\frac{2k}{3} i \sin(t + \alpha) \\ &= -\frac{2E_{ao} [F \cos(t + \alpha) - A] \sin(t + \alpha)}{H} \\ &= -\frac{2E_{ao} F}{X_d'' + X_{s2}} 0 \sin(t + \alpha) + \frac{E_{ao} A}{X_{s2}} E \sin(t + \alpha) \end{aligned} \quad (9.43)$$

Doherty and Nickle also derived formulae for (a) field current, (b) voltage across external reactance in armature circuit, (c) voltage across external reactance in field circuit, (d) voltage across the open phase, etc., which are not reproduced here.

The short-circuit torque as derived by Doherty and Nickle is as follows:-

From Park's equations,

$$\begin{aligned} T &= i_q \phi_d - i_d \phi_q \\ &= i_q (X_{ad} I_d - X_d i_d) - i_d (X_{aq} I_q - X_q i_q) \\ &= i_q X_{ad} I_d - i_d X_{aq} I_q - i_d i_q (X_d - X_q) \end{aligned} \quad (9.44)$$

Now

$$X_{ad} I_d = E_{ao} F + i_d (X_d - X_d'')$$

$$X_{aq} I_q = i_q (X_q - X_q'')$$

Substituting into (8.4.4)

$$T = \frac{2 F A E_{ao}^2}{X_{s2} + X_d''} \left\{ \sin(t + \alpha) + 3b \sin 3(t + \alpha) + 5b^2 \sin 5(t + \alpha) + \dots \right\} - \frac{E_{ao}^2}{X_{s2} + X_d''} \left[F^2 \frac{X_{s2}}{X_{s2} + X_d''} + A^2 \frac{X_{s2} - X_d''}{X_{s2}} \right] \left\{ 2 \sin 2(t + \alpha) + 4b \sin 4(t + \alpha) + \dots \right\} \quad (9.45)$$

The decrement factors are:-

$$\sigma_{ds}' = \frac{X_d + X_{s2}}{X_d' + X_{s2}} \sigma_{do}' \quad (9.46)$$

where

$$\sigma_{do}' = \frac{1}{T_{do}'} = \frac{R_{fd}}{\omega L_{fd}}$$

$$\sigma_{ds}'' = \frac{X_d'' (X_d' + X_{s2})}{X_d' (X_d'' + X_{s2})} \sigma_d'' \quad (9.47)$$

where σ_d'' is the d-axis damper winding decrement factor for three-phase short circuit.

$$\sigma_a = \frac{\frac{k^2+1}{2} r_a}{\sqrt{X_d'' X_a''}} = \frac{\frac{k^2+1}{2} r_a}{\frac{2k^2}{3} X_{s2}} = \frac{3(k^2+1)}{4k^2} \frac{r_a}{X_{s2}} \quad (9.48)$$

where $\frac{k^2+1}{2} r_a$ is the resistance of the short-circuit winding.

σ_{ds}' applies to the even-series in the field current (which is the odd-series times $\cos(t + \alpha)$, or $0 \cos(t + \alpha)$), and to a part of the odd-series in the armature current.

σ_{ds}'' applies to the even-series in the damper winding current and to a part of the odd-series in the armature current.

σ_a applies to the even-series in the armature current and the odd-series in the field and damper winding currents.

9. 6. (1) In the following are some results obtained by Doherty and Nickle from their analysis of the single-phase short circuit.

(a) Variable inductance makes a large difference in the relative magnitudes of the fundamental and harmonics, and thus also in the ratio of the amplitude of the fundamental to the peak value, both in the short-circuit current and in the open-phase voltage.

(b) The peak value is the same for salient-pole machines as for cylindrical-rotor machines, for the same direct-axis sub-transient reactance.

(c) As in the salient-pole machines the values of x_d'' and x_q'' are more nearly equal to each other than in cylindrical-rotor machines, the value of b is smaller and therefore salient-poles have an effect in reducing the relative magnitude of the harmonics with respect to the fundamental in the short-circuit current.

(d) When $b = 0$, i.e., $x_d'' = x_q''$, all harmonics disappear. When b is negative, i.e., with $x_d'' > x_q''$, the harmonics change sign.

(e) For a short circuit at $\alpha = 0$, i.e., at maximum flux enclosure or maximum flux linkages, the ratio of the voltage across the open phase after short circuit to the peak voltage before short circuit is

$$\rho_0 = (2 \frac{x_q''}{x_d''} - 1) \sin \beta$$

And at $\alpha = \frac{\pi}{2}$, $\rho_0 = \frac{x_q''}{x_d''} \sin \beta$

where β is the angle between the axis of the short-circuit winding and the axis of the open-circuited winding considered.

Therefore in salient-pole machines there is less tendency toward this high e.m.f. generation because x_d'' is relatively higher and x_q'' , on account of salient poles, is relatively lower than in cylindrical-rotor machines.

9. 7. THREE-PHASE SHORT CIRCUIT

The problem of three-phase short circuit was analyzed by Doherty and Nickle by means of constant linkage theorem. The case of negligible armature resistance is reproduced here.

$$\text{Now } i_a + i_b + i_c = 0 \quad (9.49)$$

Assuming that the short circuit occurs when the phase A is ahead of the poles by an angle α ,

$$A_d = \frac{2}{3} \left[i_a \cos(\alpha - t) + i_b \cos\left(\alpha + \frac{2\pi}{3} - t\right) + i_c \cos\left(\alpha + \frac{4\pi}{3} - t\right) \right]$$

$$\begin{aligned} A_q &= \frac{2}{3} \left[i_a \cos\left(\alpha - t - \frac{\pi}{2}\right) + i_b \cos\left(\alpha + \frac{2\pi}{3} - t - \frac{\pi}{2}\right) + i_c \cos\left(\alpha + \frac{4\pi}{3} - t - \frac{\pi}{2}\right) \right] \\ &= \frac{2}{3} \left[i_a \sin(\alpha - t) + i_b \sin\left(\alpha + \frac{2\pi}{3} - t\right) + i_c \sin\left(\alpha + \frac{4\pi}{3} - t\right) \right] \end{aligned}$$

$$\text{Then } \psi_{da} = X_d'' A_d$$

$$\psi_{qa} = X_q'' A_q$$

$$\therefore \psi_a = \psi_{ao} \cos(\alpha - t) + \psi_{da} \cos(\alpha - t) + \psi_{qa} \sin(\alpha - t) \quad (9.50)$$

$$\begin{aligned} \psi_b &= \psi_{ao} \cos\left(\alpha + \frac{2\pi}{3} - t\right) + \psi_{da} \cos\left(\alpha + \frac{2\pi}{3} - t\right) + \psi_{qa} \sin\left(\alpha + \frac{2\pi}{3} - t\right) \end{aligned} \quad (9.51)$$

$$\begin{aligned} \psi_c &= \psi_{ao} \cos\left(\alpha + \frac{4\pi}{3} - t\right) + \psi_{da} \cos\left(\alpha + \frac{4\pi}{3} - t\right) + \psi_{qa} \sin\left(\alpha + \frac{4\pi}{3} - t\right) \end{aligned} \quad (9.52)$$

By constant linkage theorem,

$$\psi_a = \psi_{ao} \cos \alpha$$

$$\psi_b = \psi_{ao} \cos\left(\alpha + \frac{2\pi}{3}\right)$$

$$\psi_c = \psi_{ao} \cos\left(\alpha + \frac{4\pi}{3}\right)$$

Substituting ψ_{da} , ψ_{qa} , ψ_a , ψ_b and ψ_c into equations (9.49) - (9.52) and solving for i_a , i_b , and i_c ,

$$\begin{aligned} i_a &= \psi_{ao} \frac{X_d'' + X_q''}{2X_d''X_q''} \cos \alpha - \frac{\psi_{ao}}{X_d''} \cos(t - \alpha) \\ &\quad - \psi_{ao} \frac{X_d'' - X_q''}{2X_d''X_q''} \cos(2t - \alpha) \end{aligned} \quad (9.53)$$

$$i_b = E_{ao} \frac{X_d'' + X_q''}{2X_d''X_q''} \cos(\alpha + \frac{2\pi}{3}) - \frac{E_{ao}}{X_d'} \cos(t - \alpha - \frac{2\pi}{3}) \\ - E_{ao} \frac{X_d'' - X_q''}{2X_d''X_q''} \cos(2t - \alpha - \frac{2\pi}{3}) \quad (9.54)$$

$$i_c = E_{ao} \frac{X_d'' + X_q''}{2X_d''X_q''} \cos(\alpha + \frac{4\pi}{3}) - \frac{E_{ao}}{X_d'} \cos(t - \alpha - \frac{4\pi}{3}) \\ - E_{ao} \frac{X_d'' - X_q''}{2X_d''X_q''} \cos(2t - \alpha - \frac{4\pi}{3}) \quad (9.55)$$

The fundamental component is proportional at each instant to the average value of the d.c. excitation. Immediately after short circuit the total d.c. excitation is $E_{ao} \frac{X_d}{X_d'}$, and under sustained short circuit this decays to E_{ao} . Hence the sustained fundamental current in the armature will be $\frac{X_d}{X_d'}$ times the initial value.

The d.c. excitation due to the main field current alone is $E_{ao} \frac{X_d}{X_d'}$. The difference between this excitation and the excitation E_{ao} , i.e.,

$$E_{ao} \frac{X_d}{X_d'} - E_{ao} = E_{ao} \frac{X_d - X_d'}{X_d'}$$

decays according to the field decrement T_d' .

Likewise the difference between the total d.c. excitation and the d.c. excitation due to the main field current alone, i.e.,

$$E_{ao} \frac{X_d}{X_d''} - E_{ao} \frac{X_d}{X_d'} = E_{ao} \frac{X(X_d' - X_d'')}{X_d'X_d''}$$

decays according to the d-axis damper winding decrement T_d'' .

Therefore the transient total d.c. excitation is

$$X_{ad} I_d (d.c.) = \frac{X_d}{X_d''} F E_{ao}$$

where

$$F = \frac{X_d'}{X_d} + \frac{X_d''}{X_d} \frac{X_d - X_d'}{X_d'} e^{-\frac{T_d'}{T_d} t} + \frac{X_d' - X_d''}{X_d'} \frac{T_d''}{T_d} e^{-\frac{T_d''}{T_d} t}$$

The fundamental component of armature current at any time after short circuit is thus F times its initial magnitude.

The d.c. component and the second harmonic of armature current decay according to the armature decrement σ_a .

Therefore the transient armature currents are as follows:-

$$i_a = \frac{E_{ao}}{X_d''} F \cos(t - \alpha) - E_{ao} e^{-\sigma_a t} \left[\frac{X_d'' + X_q''}{2X_d''X_q''} \cos \alpha - \frac{X_d'' - X_q''}{2X_d''X_q''} \cos(2t - \alpha) \right] \quad (9.56)$$

$$i_b = \frac{E_{ao}}{X_d''} F \cos(t - \alpha - \frac{2\pi}{3}) - E_{ao} e^{-\sigma_a t} \left[\frac{X_d'' + X_q''}{2X_d''X_q''} \cos(\alpha + \frac{2\pi}{3}) - \frac{X_d'' - X_q''}{2X_d''X_q''} \cos(2t - \alpha - \frac{2\pi}{3}) \right] \quad (9.57)$$

$$i_c = \frac{E_{ao}}{X_d''} F \cos(t - \alpha + \frac{2\pi}{3}) - E_{ao} e^{-\sigma_a t} \left[\frac{X_d'' + X_q''}{2X_d''X_q''} \cos(\alpha - \frac{2\pi}{3}) - \frac{X_d'' - X_q''}{2X_d''X_q''} \cos(2t - \alpha + \frac{2\pi}{3}) \right] \quad (9.58)$$

The decrement factors are found as follows:-

$$\therefore T_d' = \frac{X_d'}{X_d} T_{do}'$$

$$\therefore \sigma_a' = \frac{X_d}{X_d'} \sigma_{do}' \quad (9.59)$$

$$\text{where } \sigma_{do}' = \frac{R_{fd}}{\omega L_{fd}}$$

$$\therefore T_d'' = \frac{X_d''}{X_d'} T_{do}''$$

$$\therefore \sigma_a'' = \frac{X_d}{X_d''} \sigma_{do}'' \quad (9.60)$$

where σ_{do}'' is the d-axis open-circuit decrement factor of damper winding.

$$\sigma_a = \frac{r_a}{2X_d''X_q''} = \frac{r_a (X_d'' + X_q'')}{2X_d''X_q''} \quad (9.61)$$

where $\frac{r_a}{2X_d''X_q''}$ is the armature reactance to the direct component of armature current. It is equal to the negative-phase-sequence reactance for a sine-wave voltage.

The field current was found to be:-

$$I_{fd} = \frac{1}{X_{ad}} \left[E_{ao} + E_{ao} \frac{X_d - X_d'}{X_d'} e^{-\gamma_d t} - E_{ao} \frac{X_d - X_d'}{X_d'} e^{-\gamma_d t} \cos t \right] \quad (9.62)$$

Doherty and Nickle also analyzed the cases of (a) armature resistance not neglected, and (b) short circuit under load with armature resistance neglected, and derived formulae for the armature and field currents and the decrement factors. As the method of derivation is the same as above, they are not reproduced here.

9. 8. STARTING PERFORMANCE

The starting performance of synchronous machines has been analyzed by Putman, Linville, and Liwschitz, the Linville's method of analysis is reproduced here.

In starting there is a slip S and therefore the rotor is not stationary with respect to the rotating armature m.m.f. wave, but at a slip speed S in the backward direction. In other words the armature m.m.f. wave is rotating forward at a slip speed S with respect to the rotor. Because of the change of permeance due to the saliency of poles, the armature m.m.f. wave is not constant, but changes from a value of i_d in the d-axis to a value of i_q in the q-axis. Here it is to be noted that i_d and i_q are not components of the m.m.f., but represent the whole m.m.f. at different positions with respect to the poles.

The rotating m.m.f. wave may be resolved into two components i_f and i_b , i_f equal to $\frac{1}{2}(i_d + i_q)$ and rotating at $+S$ speed with respect to the poles, and $i_b = \frac{1}{2}(i_d - i_q)$ and rotating at $-S$ with respect to the poles.

$$\therefore i_f = \frac{1}{2}(i_d + i_q)$$

$$i_b = \frac{1}{2}(i_d - i_q)$$

The m.m.f. multiplied by the impedance and rotating speed will give the voltage. If we conceive the voltage to be composed of e_f and e_b analogously to the current, we have

$$\begin{aligned} e_f &= \frac{1}{2}(e_d + e_q) \\ &= \frac{1}{2}(i_d \cdot Z_{ad} + i_q \cdot Z_{aq}) \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} [(i_f + i_b) Z_{ad} + (i_f - i_b) Z_{aq}] \\
 &= \left[i_f \frac{Z_{ad} + Z_{aq}}{2} + i_b \frac{Z_{ad} - Z_{aq}}{2} \right] \cdot 1 \quad (9.63)
 \end{aligned}$$

$$\begin{aligned}
 e_b &= \frac{1}{2} [e_d - e_q] \\
 &= \left[i_f \frac{Z_{ad} - Z_{aq}}{2} + i_b \frac{Z_{ad} + Z_{aq}}{2} \right] (2s-1) \quad (9.64)
 \end{aligned}$$

where Z_{ad} and Z_{aq} are the impedances in the d-axis and q-axis respectively. If the resistances in the rotor circuits are neglected they become that part of the sub-transient reactance due to armature reaction.

The backward m.m.f., and therefore flux wave, rotates at speed $-S$ with respect to the poles, or at $(1-2S)$ speed with respect to the armature. Therefore conceived in the backward direction, the rotating speed is $(2S-1)$.

Assuming sine-wave voltage, then

$$\begin{aligned}
 e &= 1 = e_f + i_f (r_a + jX_l) \\
 &= i_f \frac{Z_{ad} + Z_{aq}}{2} + i_b \frac{Z_{ad} - Z_{aq}}{2} + i_f (r_a + jX_l) \quad (9.65)
 \end{aligned}$$

And

$$\begin{aligned}
 0 &= e_b + i_b [r_a + jX_l (2s-1)] \\
 &= \left[i_f \frac{Z_{ad} - Z_{aq}}{2} + i_b \frac{Z_{ad} + Z_{aq}}{2} \right] (2s-1) + i_b [r_a + jX_l (2s-1)] \\
 &= i_f \frac{Z_{ad} - Z_{aq}}{2} + i_b \frac{Z_{ad} + Z_{aq}}{2} + i_b \left[\frac{r_a}{2s-1} + jX_l \right] \quad (9.66)
 \end{aligned}$$

where r_a is the armature resistance and X_l the armature leakage reactance.

Let

$$\begin{aligned}
 Z_d &= Z_{ad} + r_a + jX_l \\
 Z_q &= Z_{aq} + r_a + jX_l \\
 Z_d' &= Z_{ad} + \frac{r_a}{2s-1} + jX_l = Z_d + \left[\frac{r_a}{2s-1} - r_a \right] \\
 Z_q' &= Z_{aq} + \frac{r_a}{2s-1} + jX_l = Z_q + \left[\frac{r_a}{2s-1} - r_a \right]
 \end{aligned}$$

Solving for i_f and i_b ,

$$i_f = \frac{Z_d' + Z_q'}{Z_d Z_q' + Z_d' Z_q} \quad (9.67)$$

$$i_b = \frac{Z_q' - Z_d'}{Z_d Z_q' + Z_d' Z_q} \quad (9.68)$$

At 50% slip these equations are indeterminate.

But here

$$i_f = \frac{2}{Z_d + Z_q}, \quad i_b = 0$$

The impedances can be determined from the equivalent circuits.

The power is

$$P = E_f \cdot i_f - i_f^2 r_a - i_b^2 r_a / (1 - 2s) \quad (9.69)$$

The torque is

$$T = \text{real of } i_f - i_f^2 r_a - i_b^2 r_a / (1 - 2s) \quad (9.70)$$

9.9. PULLING-INTO-STEP

The problem of pulling-into-step of a synchronous motor is very different to solve, because its statement involves non-linear differential equations. The integrator has been applied to solve this problem and by means of this machine Edgerton and Zak treated the case of a cylindrical rotor machine, and Edgerton and Fourmarier a salient-pole machine. But as the integrator is not comprehensive enough, many simplifying assumptions were made in the papers mentioned above. The completion of the differential analyzer at the Massachusetts Institute of Technology made possible the rapid solution of more complicated differential equations and by means of this analyzer Edgerton, Brown, Germechausen and Hamilton analyzed the pulling-into-step phenomena of both cylindrical rotor and salient-pole machines by introducing into the equation the time constant of the field circuit with short-circuited armature. The differential analyzer completed at the University of Pennsylvania is still more comprehensive, so that still fewer assumptions are necessary and the major electrical transients and mechanical forces may be taken into account. Shoutts, Crary and Lauder analyzed this problem based upon the generalized two reaction theory of synchronous

machine performance, using equations presented by Park. Therefore the results in their paper are the most accurate and reliable to date.

Criteria for pulling-into-step were presented in their paper for the performance of motors when the field excitation is applied at the most favourable and at the most unfavourable angles.

The criterion for the most favourable angle is:-

$$T_L < \left\{ \frac{2 E_{d'(max)}^2}{\pi X_{d'}} + \frac{3}{8} e^2 S_0 \left[\frac{X_{d'} - X_{d''}}{X_{d'} X_{d''}} T_{d''} - \frac{X_q - X_{q''}}{X_q X_{q''}} T_{q''} \right] - 2 f H S_0^2 - \left(\frac{E_{d'(max)}}{X_{d'}} \right)^2 r_a \right\} \quad (9.71)$$

where T_L = mechanical load torque

$E_{d'(max)}$ = voltage back of transient reactance corresponding to the maximum field flux linkages during the induction motor cycle. (Voltage back of transient reactance

$$E_{d'} = \frac{X_{afd}}{X_{fd}} I_{fd})$$

S_0 = slip at $\delta = 0$ when machine is running out of synchronism.

H = inertia constant

$$= \frac{0.231 \times WR^2 \times (r.p.m.)^2}{10^6 \times \text{base kVA}}$$

In eq. (9.71) the first term inside the brackets results from the synchronous component of transient torque, the second term from the component of torque contributed by the amortisseur winding, the third term from the kinetic energy component of torque, and the fourth term from the armature resistance losses.

Both $E_{d'(max)}$ and S_0 can be calculated for practical purposes on the assumption of constant slip. This formula is adopted particularly to motors having long field-time-constants, as it assumes constant field flux-linkages during the synchronizing period.

For small machines with short field-time-constants the terms

$$\frac{2 E_d'(\max) e}{\pi X_d'} \quad \text{and} \quad \left(\frac{E_d'(\max)}{X_d'} \right)^2 r_a$$

can be replaced by

$$\frac{2 E_d e}{\pi X_d} \quad \text{and} \quad \left(\frac{E_d}{X_d} \right)^2 r_a$$

respectively,

where E_d = direct-axis voltage corresponding to the field excitation .

$$\therefore T_L < \left\{ \frac{2 E_d e}{\pi X_d} + \frac{3}{8} e^2 S_o \left[\frac{X_d' - X_d''}{X_d' X_d''} T_d'' - \frac{X_q - X_q''}{X_q X_q''} T_q'' - 2 f H S_o^2 - \left(\frac{E_d}{X_d} \right)^2 r_a \right] \right\} \quad (9.72)$$

If there is an appreciable difference between the load torque that can be pulled into step on the basis of constant field flux linkages (equation (9.71)) as compared with that determined by using equation (9.72), a simple step-by-step calculation can be made to determine the approximate time-angle relation for the motor during the synchronizing period and a corresponding correction made as follows:-

$$T_L < \left\{ k_1 \left(\frac{2 E_d'(\max) e}{\pi X_d'} \right) + \frac{3}{8} e^2 S_o \left[\frac{X_d' - X_d''}{X_d' X_d''} T_d'' + \frac{X_q - X_q''}{X_q X_q''} T_q'' \right] - 2 f H S_o^2 - k_1 \left(\frac{E_d'(\max)}{X_d'} \right)^2 r_a \right\} \quad (9.73)$$

where k_1 is a correction factor to take into account the increase in the field flux linkages during the synchronizing period.

The criterion for the most unfavourable angle is:-

$$S_{av} < k_2 \sqrt{\frac{E_d e}{\pi f H X_d}} \quad (9.74)$$

where S_{av} = average slip

k_2 = ratio between the average slip S_{av} and the total slip pulsation S_p .

From the differential analyzer results, $k_z = 0.55$ to 0.60 . The value of slip s_{av} is approximated by the average slip during the last slip cycle.

It was pointed out in that paper that these criteria checked the results obtained on the differential analyzer with good accuracy.

CHAPTER X

SOME NOTES ON THE DESIGN OF SALIENT-POLE SYNCHRONOUS MACHINES

10.1. ADVANTAGES OF SALIENT-POLE SYNCHRONOUS MACHINES

To permit taking full advantage of separate excitation with the large number of field ampere-turns required, it is desirable that the field copper space be a maximum. Therefore very deep field slots are required, and for maximum space factor, only one slot per pole should be used. Hence the salient-pole field winding is preferable to the phase-wound field for a synchronous machine. For very-high-speed machines, mechanical stresses prevent the use of salient-poles, but otherwise they are almost universally employed.

Because of its greater excitation capacity, the salient-pole synchronous machine may employ a relatively large airgap, which is mechanically desirable, without detriment to the electrical characteristics.

Salient-pole synchronous motors have the advantage over induction motors in that greater airgap and more poles can be used. In order to reduce the magnetizing kVA_r of induction motors, it is necessary to use small airgaps of the order of 0.25% of the pole pitch. Mechanical limitations prevent the use of airgaps smaller than .1% of the diameter, so that induction motors with more than 8 poles are definitely handicapped. The normal design of the salient-pole synchronous motors permits a natural airgap of the order of $1\frac{1}{2}$ to 2% of the pole pitch and mechanical limitations are not reached until the motor has 60 poles or more. For this reason, the lower the speed, the more favourable are the synchronous motor characteristics in comparison with those of the induction motor.

The large airgap makes possible the use of larger stator slots with consequent improvement of space factor and efficiency. This comes about because the slot-pitch must be related to the length of the airgap to avoid high-tooth-frequency iron losses. Furthermore, small high-frequency flux pulsations cannot cross a large airgap, hence thicker laminations can be used in the pole shoes.

10.2. NORMAL DESIGN FEATURES OF SALIENT-POLE SYNCHRONOUS MOTORS

While both generators and motors can be of the salient-pole type, there are no special difficulties and considerations in the design of synchronous generators, whereas in the design of salient-pole synchronous motors the starting torque is usually the main consideration. To meet special requirements special designs are usually made to obtain the desired torque/speed characteristics.

In this chapter only brief references to the design features of salient-pole synchronous motors can be given.

While the rotating member may be either the field or the armature, the rotating field is normal, with centrifugal stresses and simplicity dictating that it be inside the armature.

This normal design provides space in the pole face for an amortisseur winding with ample heat capacity to enable the motor to start most industrial loads. The conclusion is that the normal type of synchronous motor which is best for steady load operation has the following design characteristics:-

- (1) Separate d.c. excitation.
- (2) Internal revolving field.
- (3) Laminated salient field poles.
- (4) Airgap equal to $1\frac{1}{2}$ to 2% of the pole pitch.
- (5) Amortisseur winding section of sufficient heat capacity to enable the motor to safely start the load.
- (6) Open armature slots, somewhat fewer in number than those of an equivalent induction motor.

10.3. REQUIREMENTS FOR SALIENT-POLE SYNCHRONOUS MOTORS

High speed motors usually have low reactance and consequently high starting torque and current. Because of the heat capacity of the motor it is sometimes necessary to reduce the starting torque and therefore limit the starting current. Apart from the design of the motor, there are several methods of control to reduce the starting torque and current. They are:-

- (1) Auto-transformers to reduce the applied voltage.
- (2) Series reactors and resistors to absorb a part of the applied voltage.
- (3) Double-circuit stator winding -- In starting only one circuit of the stator winding is connected to the applied voltage so as to increase the impedance of the motor and therefore limit the starting current.

Low speed motors usually have high reactance, and consequently low starting and pull-in torque. It is therefore sometimes necessary to increase the starting torque by such methods as:-

- (1) Oversize high-torque motor.
- (2) Normal motor started on over-voltage.

- (A) Autotransformer.
- (B) Applying line voltage to only a part of the primary winding.
- (3) Delta-Star starting.
- (4) Tapped-winding starting.
- (5) Starting with capacitor across motor terminals -- improving power factor.
- (6) Use of a variable resistor in field circuit.
- (7) The super-synchronous motor.
- (8) Phase-wound damper winding.

The phase-wound damper winding is a special design of synchronous motor and will be discussed later.

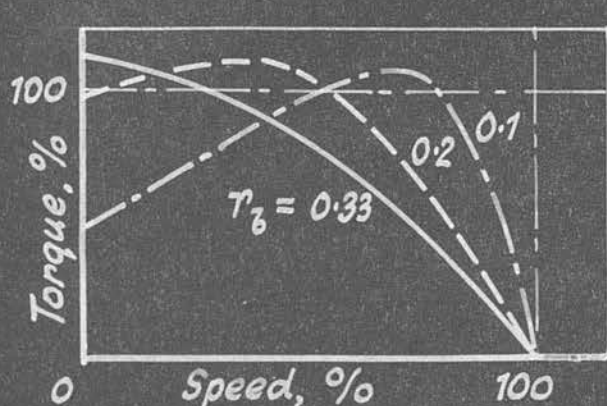
10.4. DESIGN OF DAMPER WINDINGS

From above it can be seen that the starting torque is determined by the field and damper windings. As the field is usually designed according to the rating of the machine, the damper winding remains the only important consideration in design to meet the requirements of starting torque and pull-in torque. Here we shall discuss the design of damper windings qualitatively.

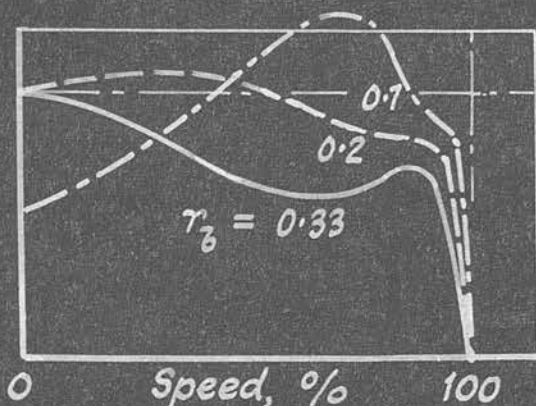
The effect of damper-winding resistance on torque is the same as that of rotor resistance in induction motors. The smaller the resistance the nearer the speed for maximum torque is to synchronous speed, and the larger is the pull-in torque. The following curves are taken from a paper by Shutt roughly showing the effect of change in damper winding resistance on torque of a 2,000 H.P., 150 r.p.m., 100% P.F., synchronous motor. From the torque/speed curves and the requirements of starting and pull-in torques the proper resistance of the damper windings can be chosen. The resistance of the damper bars depends on the material and the size of the bars.

In order to get high torque during the starting period, the reactance of the damper windings should be as low as possible if the ratio of specific permeance to effective specific resistance were always constant. But as for high-reactance damper windings owing to the skin effect the ratio is not constant and becomes larger as the slip gets smaller, there is usually a larger pull-in torque for motors with high-reactance than for those with low-reactance damper windings. Therefore according to the requirements high-reactance damper windings are sometimes necessary.

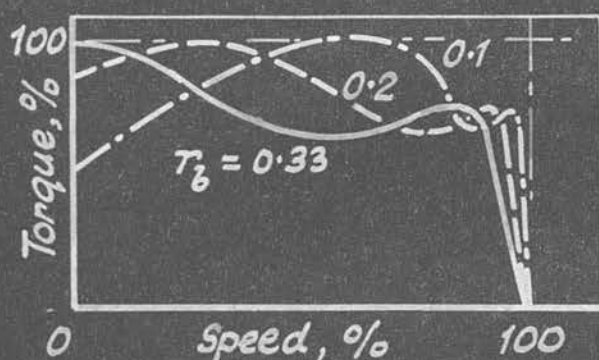
Low-reactance damper windings include all windings which are made up of round or rectangular bars of such size that skin effect



(A) FIELD CIRCUIT OPEN



(B) FIELD CLOSED THROUGH NORMAL STARTING RESISTOR.



(C) STARTING RESISTOR VARIED IN SAME RATIO AS DAMPER WINDING RESISTANCE.

FIG. 10.1. Effect on torque of change in damper winding resistance.

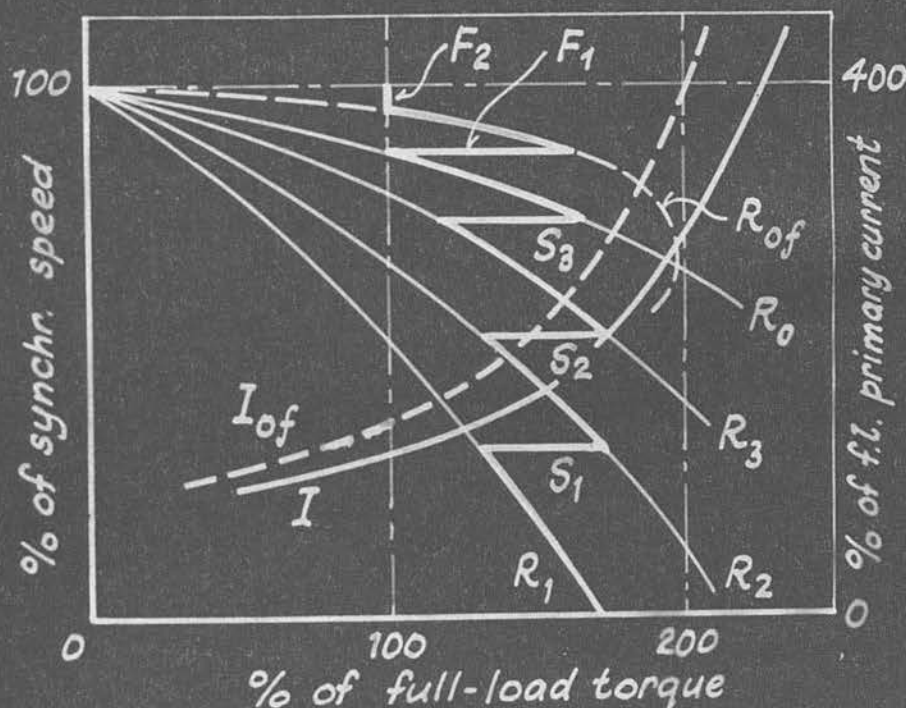


FIG. 10.2. Speed/torque & current/torque curves for syn. motor with phase-connected damper winding.

has negligible effect on the distribution of the current in the bar. Such windings are so arranged in the pole shoe that the ratio of specific permeance to effective specific resistance of the winding as a whole is only slightly affected as the motor accelerates. Shutt showed that the specific permeance and effective specific resistance for round and square bars are practically unchanged over the slip range 60 to 3.5 ϵ/s .

High-reactance damper windings include all forms of double deck windings, such as those having two distinct bars placed one over the other and all windings made entirely or in part of L bars or inverted T bars as well as plain rectangular bars of such a shape and material that they are subject to an appreciable skin effect. Moreover, they include all single deck windings which are so arranged that the characteristics of the winding change noticeably as the motor comes up to speed. Shutt showed that the ratio of specific permeance to effective specific resistance can be increased six or seven times over the slip range of 60 to 3.5 ϵ/s for deep rectangular bars and double-deck round bars.

From above it is seen that in order to obtain the highest ratio of torque to kVA throughout the starting period, it is necessary to have the rotor circuits of high resistance and low reactance. But very high resistance damper windings have the serious practical objections that they produce low torque at the higher speeds unless the motor has a low stator reactance.

When a high ratio of torque to kVA is required, especially during the early part of the starting period, and when the starting torque itself must be high with a relatively low kVA , low-reactance damper windings are suitable. They may be employed for low-speed motors with 40 to 50% pull-in torque accompanied with a starting torque as high as 75%. Most high-speed motors will produce 100% pull-in torque with 150 to 175% starting torque with low-reactance dampers.

The double squirrel-cage type of damping winding in salient-pole machines is particularly suitable to applications requiring relatively high pull-in torques with low starting kVA , the starting torque not materially exceeding the pull-in torque. The double-cage form of damper winding is less effective in low- than in high-speed motors because of the limited space available in the low-speed pole head for submerging the high reactance element, though it has been used rather commonly on low speed synchronous motors which require nearly a uniform torque throughout the starting period,

In general, for synchronizing with a large inertia, either a phase-wound or a cage damper can be designed with very low resistance to develop high torque at very low slip. It seems to be character-

istic of most flywheel loads that the load at starting is not high, hence the moderate starting torque obtained with a low resistance squirrel cage of normal reactance is usually adequate for starting loads of this type.

10.5. SYNCHRONOUS MOTOR WITH PHASE-CONNECTED DAMPER WINDING FOR HIGH-TORQUE LOADS

For the application of low-speed synchronous motors to industrial loads requiring both high starting torque and high pull-in torque with moderate starting current, a form of synchronous motor with phase-connected damper winding has been contrived giving a high torque/ kVA ratio with simple mechanical construction. This motor is of salient-pole field design with the very desirable operating features of that type. Starting characteristics resembling those of the slip-ring induction motor are attained through the use of a special form of damper winding, in which the bars are of low resistance and, instead of being joined to end rings to form the usual cage construction, are phase-connected and brought out through slip rings to an external starting resistor which is varied by suitable control.

To develop the induction motor torques to the greatest extent the construction throughout is directed toward low reactance. Thus the stator employs a large number of slots with more coils and fewer turns per coil than are commonly used in ordinary low-torque machines. The field is open-circuited at starting, thereby eliminating an undesirable low-power-factor component in the primary current ordinarily reflected from the circulation of current in the highly reactive field circuit when closed on a resistor of the proportions commonly used.

The speed/torque and current/torque curves for such a motor is shown in Fig. 10.2.

In Fig. 10.2 the heavy line is the speed/torque curve.

S_1 , S_2 , and S_3 show the sudden changes of torque when the resistances in the damper winding are cut out. F_1 shows the closing of field circuit without excitation and F_2 shows the introduction of excitation leading to synchronising.

10.6. EXAMPLE OF THE CALCULATION OF SYNCHRONOUS REACTANCES

The machine is an alternator designed by Bruce Peebles & Company, Limited. It is three-phase, 50 cycles, 16 poles, 375 r.p.m., 5325 KVA, 0.8 P.F., 6,000 V terminal voltage and 512 amperes terminal current.

The physical dimensions that are needed in the calculation of

reactances are as follows.

$$R_a = 1275 \text{ m.m.} = 50.2 \text{ in.}$$

$$l = 550 \text{ m.m.} = 21.65 \text{ in.}$$

$$\gamma = 502 \text{ m.m.} = 19.8 \text{ in.}$$

$$g' = 12.5 \text{ m.m.} = .491 \text{ in.}$$

$$g = 12.5 \times 1.13 = 14.1 \text{ m.m.} = .555 \text{ in.}$$

$$\text{Maximum gap} = 21 \text{ m.m.} = .83 \text{ in.}$$

$$\therefore \frac{\text{Max. gap}}{\text{Min. gap}} = \frac{.83}{.555} = 1.48$$

$$\frac{g}{\gamma} = \frac{.555}{19.8} = 0.028$$

$$\alpha = .672$$

The data for armature winding that are needed in the calculation of reactances are as follows:-

$$\text{No. of slots} = 240$$

$$q = 5$$

$$\text{Parallel circuits} = 4$$

$$\text{Conductors per slot} = 2 \times 6 = 12$$

$$\text{Coil throw} = 1 - 13$$

$$\therefore n = \frac{5 \times 12}{2 \times 4} = 7.5$$

$$k_p = 0.951$$

$$k_d = 0.956$$

$$h_1 = 74.5 \text{ m.m.} = 2.93 \text{ in.}$$

$$h_2 = 7.2 \text{ m.m.} = .284 \text{ in.}$$

$$b_s = 14.5 \text{ m.m.} = .57 \text{ in.}$$

$$b_t = 18.9 \text{ m.m.} = .745 \text{ in.}$$

$$g' = 12.5 \text{ m.m.} = .491 \text{ in.}$$

$$l_{e2} = 3.74 \text{ in.}$$

$$l_{e1} = 7.12 \text{ in.}$$

Now from Wieseeman's curves,

$$C_1 = 1.04$$

$$C_{d1} = .875$$

$$C_{q1} = .439$$

$$\begin{aligned} \therefore X_{ad} &= \frac{512}{\frac{6000}{\sqrt{3}}} \times 2\pi \times 50 \times \frac{76.56}{\pi 10^8} \frac{(k_p k_d n)^2 C_{d, Ral}}{9} \\ &= \frac{512}{3465} \times 2\pi \times 50 \times \frac{76.56 (.951 \times 956 \times 7.5)^2 \times 50.2 \times 21.65}{\pi 10^8 \times .555} C_{d,} \end{aligned}$$

$$= 1.025 C_{d,}$$

$$= 1.025 \times .875$$

$$= .895$$

$$X_{aq} = 1.025 C_{q,}$$

$$= 1.025 \times .439$$

$$= .450$$

From Fig. 7.5,

$$k_x = .85$$

$$C_x = \frac{.85}{.951^2 \times .956^2} = 1.022$$

$$\begin{aligned} \therefore \lambda_i &= \frac{20 \times 1.022}{15} \left(\frac{.284}{.57} + \frac{2.93}{1.71} + .2 + \frac{.07 \times .745}{.491} \right) \\ &= 3.43 \end{aligned}$$

$$\lambda_B = 0$$

$$\lambda_e = \frac{4}{21.65} (7.48 + 7.12) = 2.7$$

$$\therefore X_i = \frac{512}{\frac{6000}{\sqrt{3}}} \times 2\pi \times 50 \times \frac{6 \times 10^{-8}}{\pi} \frac{(.951 \times 956 \times 7.5)^2 \times 16 \times 21.65 \times (3.43 + 2.7)}{9}$$

$$= 0.01425 \times 6.13$$

$$= .0873$$

$$X_d = .895 + .0873$$

$$= .982$$

$$X_q = .45 + .0873$$

$$= .537$$

Now from flux plotting and Fourier's Harmonic Analysis,

$$C_i = 1.079$$

For the sake of simplicity, Kilgore's expression for pole shape coefficients are used instead of Rankin's more accurate expressions.

$$C_m = \frac{\pi \times .672 + \sin\left(\frac{\pi \times .672}{2}\right)}{4 \sin\left(\frac{\pi}{2}\right)} = .852$$

$$\therefore C_d = .852 \times 1.079 = .918$$

which is a little bigger than that from Wieseman's curve

$$\begin{aligned} \therefore X_{ad} &= 1.025 C_d \\ &= 1.025 \times .918 \\ &= .942 \end{aligned}$$

$$C_q = \frac{4 \times .672 + 1}{5} - \frac{\sin\left(\frac{\pi \times .672}{2}\right)}{\pi} = .465$$

which is also a little bigger than that from Wieseman's curve

$$\begin{aligned} \therefore X_{aq} &= 1.025 C_q \\ &= 1.025 \times .465 \\ &= .475 \end{aligned}$$

$$\begin{aligned} \therefore X_d &= .942 + .0873 \\ &= 1.029 \end{aligned}$$

$$\begin{aligned} X_q &= .475 + .0873 \\ &= .562 \end{aligned}$$

10.7. CONCLUSION

In general, the design of a synchronous machine is based on the rating. The per-unit direct-axis synchronous reactance is chiefly dependent on the airgap, the larger the airgap the smaller the x_d .

The ratio of x_q to x_d depends on the pole shape. Since the pole shape is subject only to changes within a restricted range, x_q usually bears a certain ratio to x_d and does not depart very much from it.

The transient and sub-transient reactances depend upon the leakage reactances of the field winding and the damper winding. But in designing the damper winding the chief consideration is the starting and pull-in torques which are determined by the specification to be met. Low damper-winding reactance results in low

sub-transient reactances and high damper-winding reactance results in high sub-transient reactances. The resistance of the damper winding has effect in the decrement of the short-circuit current, the larger the resistance the larger the decrement factor.

From the design data the machine constants can be calculated and from tests of the machine these constants can also be obtained. The performance of a machine depends on the machine constants and can be expressed in terms of them.

The development of the Two-Reaction Theory introduces more constants so that the machine performance can be now more accurately and exactly expressed than before with less assumptions and approximations. The range of application of synchronous machines is enlarged and special machines can be designed to meet special requirements.

In the following is a table of typical values of some of the machine constants of various kinds of synchronous machines reproduced from a paper by Park and Robertson.

TYPICAL VALUES OF REACTANCE

	Synchronous Motor		Syn. con- denser	Water- wheel generator	Turbo-alternator	
	High- speed	Low- speed			Solid rotor	Laminat ^d rotor
x_d	0.65-0.9 0.8	0.8-1.5 1.1	1.6	0.6-1.25 1.0	1.15	1.15
x_q	0.5-0.7 0.65	0.6-1.1 0.8	1.0	0.4-0.8 0.65	1.0	1.0
x'_d	0.15-0.35 0.25	0.4-0.7 0.5	0.4-0.5	0.2-0.45 0.35	0.15-0.25	0.15-0.25
x'_q	0.5-0.7 0.65	0.6-1.1 0.8	1.0	0.4-0.8 0.65	0.15-0.25	1.0
x''_d	0.1-0.25 0.18	0.25-0.45 0.35	0.25-0.3	0.15-0.35 0.25	0.08-0.15	0.08-0.12
x''_q	0.12-0.3 0.2	0.3-0.5 0.4	0.25-0.35	0.4-0.8 0.65	0.1-0.2	0.2-0.3
x_2	0.11-0.25 0.19	0.25-0.5 0.35	0.25-0.32	0.25-0.6 0.45	0.08-0.13	0.11-0.21
* x_0	0.02-0.15	0.04-0.27	0.04-0.1	0.02-0.21	0.01-0.08	0.01-0.08
T'_{do}	2-4	2-4	5-7	3-6	4-7	4-7

Reactances x in per-unit values. Time-constant T in sec.
 * x_0 varies from 15 to 60 per cent. of x''_d depending on the winding pitch.

A C K N O W L E D G E M E N T S

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N O M E N C L A T U R E

The quantities given in the following, if they have per-unit values, represent the per-unit values as well as the ampere-length values, except for those in the conversion formulae in Chapter V, where per-unit values are expressed in terms of ampere-length values, and for clarity bold-face type is used for ampere-length quantities.

For voltages, currents, reactances, resistances, and flux linkages, in general small letters e , i , x , r , and ψ are used for the armature circuit while capital letters, E , I , X , R , and Ψ are used for the rotor circuits.

Subscripts are freely used, but in general the following subscripts have definite and consistent significance :-

- a, b, c , or A, B, C --- armature phases.
- f --- field-winding circuit.
- n, k, x --- damper-winding circuits. n is the general term representing 1, 2, ...; k refers to a damper-winding circuit other than n ; and x refers to the 100 per cent-pitch circuit which is used as a base.
- d, q, o --- direct, quadrature, and "zero" axes.
- o --- base quantities.
- $'$ --- transient quantities.
- $"$ --- sub-transient quantities.
- Δ --- increment.

- A = armature reaction m.m.f. in general.
- A_d = direct-axis component of armature reaction m.m.f.
- A_q = quadrature-axis component of armature reaction m.m.f.
- A_o = flat-topped armature reaction m.m.f. due to rated current.
 $= 1.5 i_{ao} n k_p k_d$
- A_i = maximum fundamental armature reaction m.m.f. due to rated current.
 $= \frac{6}{\pi} i_{ao} n k_p k_d = 2.7 i_o n k_p k_d$
- A_{if} = field ampere-turns required to balance armature reaction m.m.f. at rated current and at zero power factor.
- A = symbol for a certain function.
- a_{end} = cross-sectional area of damper bar end-ring in the direct axis.
- a_{enq} = cross-sectional area of damper bar-end-ring in the quadrature-axis.
- a_{bn} = cross-sectional area of n^{th} damper bar
- B_{max} = maximum flux density of field at rated voltage and no load.
- B_i = maximum fundamental flux density of field at rated voltage and no load.
- $B_{ga(max)}$ = maximum fundamental flux density due to rated armature current.
- $B_{gf(max)}$ = maximum fundamental flux density due to field current I_{fd0}
 $= B_{ga(max)}$

- b_n = dimension of the pole, Fig. 7.7.
 b_p = dimension of the pole, Fig. 7.7.
 b_s = dimension of armature slot, Fig. 7.3.
 b_t = dimension of armature slot, Fig. 7.3.
 b = a symbol for $\frac{\sqrt{x_q''} - \sqrt{x_d''}}{\sqrt{x_q''} + \sqrt{x_d''}}$
 C = a constant in general.
 $\frac{2}{\pi} C_0$ = ratio of average flux density to the maximum flux density with the machine excited by the field.
 C_1 = ratio of maximum fundamental to actual maximum airgap flux density produced by the field.
 C_{d1} = ratio of maximum fundamental to actual maximum airgap flux density produced by a sine wave armature reaction m.m.f. in the direct axis.
 C_{q1} = ratio of maximum fundamental airgap flux density produced by a sine wave armature reaction m.m.f. in the quadrature axis to the actual maximum airgap flux density that would be produced if the airgap were uniform and equal to the minimum gap.
 C_3 = ratio of third harmonic to fundamental of airgap flux density produced by the field.
 C_{d3} = ratio of third harmonic to fundamental of airgap flux density produced by a sine-wave m.m.f. in the direct-axis.
 C_{q3} = ratio of third harmonic to fundamental of airgap flux density produced by a sine-wave m.m.f. in the quadrature-axis.
 C_m = ratio of fundamental airgap flux density by a sine-wave armature m.m.f. in the direct axis to that by the field of the same maximum m.m.f.
 $C_x = \frac{k_x}{k_p k_d}$ where k_x is a reduction factor of slot reactance due to chording.
 $D(p)$ = a certain operational function.
 D_r = rotor diameter.
 D_e = distance between damper bar end-rings.
 $\frac{2}{\pi} D_{don}$ = the factor by which the maximum airgap flux density must be multiplied to obtain the average density within the damper circuit of span $2y_{nd}$ with the machine excited by the damper circuit of that span in the direct axis.
 $\frac{2}{\pi} D_{qon}$ = analogous to $\frac{2}{\pi} D_{don}$ in the quadrature-axis.
 D_{din} = the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by the n th additional rotor circuit in the direct axis.
 D_{qin} = analogous to D_{din} in the quadrature-axis
 d_b = depth of damper bars.
 d_r = dimension of damper bar, Fig. 7.8.
 d_{sr} = dimension of damper bar. Fig. 7.8.
 E_{fd} = field voltage.
 $E_{nd} (E_{1d}, E_{2d}, \dots)$
 = direct-axis additional rotor circuit voltage.

E_{nq} (E_{1q}, E_{2q}, \dots)
 = quadrature-axis additional rotor circuit voltage.
 E
 = symbol for an even series.
 e
 = armature voltage in general.
 e_a, e_b, e_c
 = phase terminal voltages.
 e_{ab}, e_{bc}, e_{ca}
 = line-to-line voltages.
 e_n
 = nominal open-circuit voltage.
 e'_n
 = fictitious nominal open-circuit voltage if x_q were equal to x_d .
 e''_n
 = fictitious nominal open-circuit voltage if x_d were equal to x_q .
 e_d
 = direct-axis excitation voltage.
 e_q
 = quadrature-axis excitation voltage.
 e'_d
 = voltage back of transient reactance
 $= \frac{X_{td}}{X_{td} + j\omega L_{td}} I_{td}$
 $e'_{d(max)}$
 = voltage back of transient reactance corresponding to the maximum field flux linkages during the induction motor cycle.
 e
 = airgap voltage.
 e_t
 = terminal voltage.
 e_o
 = r.m.s. rated voltage.
 e_{ao}
 = peak value of phase voltage.
 e_1
 = positive phase-sequence voltage.
 e_2
 = negative phase-sequence voltage.
 e_o
 = zero phase-sequence voltage
 $= \frac{1}{3} (e_a + e_b + e_c)$
 e_{Id}
 = reactive voltage induced by direct-axis flux produced by armature reaction m.m.f.
 e_{Iq}
 = reactive voltage induced by quadrature-axis flux produced by armature reaction m.m.f.
 e_{td}
 = direct-axis component of terminal voltage.
 e_{tq}
 = quadrature-axis component of terminal voltage.
 Δe_d
 = increment in direct-axis excitation voltage.
 Δe_q
 = increment in quadrature-axis excitation voltage.
 e_f
 = forward voltage.
 e_b
 = backward voltage.
 F
 = a symbol for a certain function.
 F_g
 = field winding ampere-turns per pole at no load and rated voltage.
 F_{st}
 = empirical coefficient determined by Wright to obtain saturated values of reactances from unsaturated values.
 $= 0.88$
 f
 = frequency.
 $G(p)$
 = operational function.
 g
 = effective minimum airgap (effective airgap at pole centre).

g' = actual minimum airgap.
 g_y = gap-reluctance mathematically expressed in terms of pole-shape dimensions as a per-unit value of the effective minimum gap g .
 g_d = g_y in the region $0 < y_d < \alpha_n$
 g_q = g_y in the region $\alpha_n < y_d < 1$
 g_η = a symbol for $1 + (\rho - 1) (\alpha_\eta / \alpha)^2$
 H = a symbol for $(x_d'' + x_q'') + (x_d'' - x_q'') \cos 2(\tau + \alpha)$
 H = inertia constant.
 $h_1, h_2, h_3,$
 = dimensions of an armature two-layer-winding slot, Fig. 7.3.
 $h_p, h_{f1}, h_{f2}, h_h,$
 = dimensions of the pole, Fig. 7.7.
 I_{fd} = field current.
 I_{fdo} = unit field current.
 I_{fdn} = field current at no load and rated voltage.
 I_{nd} (I_{1d}, I_{2d}, \dots)
 = direct-axis additional rotor circuits current.
 I_{nq} (I_{1q}, I_{2q}, \dots)
 = quadrature-axis additional rotor circuits current.
 I_{xdo} = unit current of x th additional rotor circuit (100% pitch) in the direct-axis and also unit current of all additional rotor circuits in both axes.
 I_d = equivalent direct-axis rotor circuits current.
 I_q = equivalent quadrature-axis rotor circuits current.
 \hat{I}_{fdm} = magnitude of the vector form of I_{fd} .
 \hat{I}_{ndm} = magnitude of the vector form of I_{nd} .
 \hat{I}_{nqm} = magnitude of the vector form of I_{nq} .
 i = armature current in general.
 i = peak value of armature current.
 $i_a, i_b, i_c,$
 = armature phase currents.
 i_d = direct-axis armature current component.
 \hat{i}_{dm} = magnitude of the vector form of i_d .
 i_q = quadrature-axis armature current component.
 \hat{i}_{qm} = magnitude of the vector form of i_q .
 i_o = r.m.s. rated armature current.
 i_1 = positive phase-sequence current.
 i_2 = negative phase sequence current.
 i_o = zero phase-sequence current
 $= \frac{1}{3} (i_a + i_b + i_c)$
 i_{ao} = peak value of rated armature current=unit armature current.
 i' = transient current.
 i'' = sub-transient current.
 $\Delta i'$ = $i' - i$ = difference between transient current and steady-state current.
 $\Delta i''$ = $i'' - i'$ = difference between sub-transient current and transient current.

$(\Delta i')_0$ = initial value of $\Delta i'$.

Δi_d = increment in direct-axis component current.

Δi_q = increment in quadrature-axis component current.

i_f = forward current.

i_b = backward current.

K_ϕ = ratio of total flux per pole to the total fundamental flux per pole with the machine excited by the direct-axis field winding.

k_p = pitch factor.

k_d = breadth factor.

k_n = combined pitch and breadth factor ($= k_p k_d$) of the n th harmonic.

k_x = reduction factor of slot reactance due to chording.

k_{x0} = coefficient for zero phase-sequence reactance.

k_{ad} = a coefficient equal to $\frac{4 \cdot C_{d1}}{\pi \cdot C_1}$

k_1 = correction factor in the equation of the criterion for pulling-into-step.

k_2 = ratio between the average slip S_{av} and slip pulsation S_p .

k = a coefficient equal to 1.0 or 1.73.

L_d = total armature inductance in the direct-axis.

L_q = total armature inductance in the quadrature axis.

L_{ad} = armature reaction inductance in the direct-axis.

L_{aq} = armature reaction inductance in the quadrature-axis.

L_{a0} = unit armature inductance = $\frac{e_{a0}}{2 \pi f i_{a0}}$

L_{ffd} = total field inductance.

L_{nnd} * (L_{11d} , L_{22d} , ...) = total inductance of an additional rotor circuit in the direct axis.

L_{nnq} (L_{11q} , L_{22q} , ...) = total inductance of an additional rotor circuit in the quadrature axis.

L_{afd} = mutual inductance between armature and field.

L_{and} (L_{a1d} , L_{a2d} , ...) = mutual inductance between armature and additional rotor circuit in the direct axis.

L_{anq} (L_{a1q} , L_{a2q} , ...) = mutual inductance between armature and additional rotor circuit in the quadrature axis.

L_{fnd} (L_{f1d} , L_{f2d} , ...) = mutual inductance between field and additional rotor circuit in the direct axis.

L_o = zero phase-sequence inductance.

L_{nkd} = mutual inductance between n th and k th additional rotor circuits in the direct-axis.

L_{nkq} = mutual inductance between n th and k th additional rotor circuits in the quadrature axis.

- L_l = armature leakage inductance.
 L_{H1} = field inductance due to airgap flux.
 L_{H2} = field inductance due to pole-tip and pole-body leakage.
 L_{gnd} = direct-axis damper bar inductance due to airgap flux.
 L_{gna} = quadrature-axis damper bar inductance due to airgap flux.
 L_{bnd} = direct-axis damper bar inductance due to slot leakage.
 L_{bna} = quadrature-axis damper bar inductance due to slot leakage.
 L_{end} = direct-axis damper bar inductance due to end-ring leakage.
 L_{ena} = quadrature-axis damper bar inductance due to end-ring leakage.
 l'_d = static transient inductance in the direct axis.
 l'_q = static transient inductance in the quadrature axis.
 l' = static transient inductance in any position.
 $l' = \frac{1}{2} (l'_d + l'_q) + (l'_d - l'_q) \cos 2\gamma$
 l = stacked length of machine.
 l_h, l_p = dimensions of the pole, Fig. 7.7.
 l_{e1} = extension of bent section of end-winding.
 l_{e2} = length of straight section of end-winding.
 l_{end} = length of end-ring between n th bars in the direct-axis.
 l_{ena} = length of end-ring between n th bars in the quadrature-axis.
 l_{bn} = length of n th damper bar.
 N_{fd} = field turns per pole.
 n = armature turns per phase per pole.
 n = order of harmonic of m.m.f. wave.
 O = symbol for an odd-series.
 P = airgap permeance.
 P_s^n = airgap permeance to sine m.m.f. distribution relative to the pole axis of the n th harmonic.
 P_c^n = airgap permeance to cosine m.m.f. distribution relative to the pole axis of the n th harmonic.
 P_o^n = zero harmonic (average value) of the permeance curve.
 P_z^n = second harmonic of the permeance curve.
 P_m^n = permeance distribution where n denotes the order of m.m.f. harmonic and m denotes that of the permeance wave.
 P = power.
 P_i = number of poles.
 P_p = pole pitch.
 P_s = synchronizing power = $\frac{dP}{d\delta}$
 P_2 = measured power in the test for x_2 .
 p = $\frac{d}{dt}$
 q = slots per phase per pole.

R_{fd} = total field resistance.
 R_{mnd} (R_{11d} , R_{22d} , ...) = total resistance of an additional rotor circuit in the direct-axis.
 R_{mnq} (R_{11q} , R_{22q} , ...) = total resistance of an additional rotor circuit in the quadrature axis.
 R_{bnd} = resistance of two damper bars forming one additional rotor circuit in the direct axis.
 R_{bnq} = resistance of two damper bars forming one additional rotor circuit in the quadrature axis = R_{bnd}
 R_{end} = resistance of end-rings of additional rotor circuit in the direct-axis.
 R_{enq} = resistance of end-rings of additional rotor circuit in the quadrature axis.
 R_{nkd} = mutual resistance between n th and k th additional rotor circuits in the direct-axis.
 R_{nkq} = mutual resistance between n th and k th additional rotor circuits in the quadrature-axis.
 R_a = radius of armature.
 R_f = radius at rotor surface.
 r_a = armature resistance.
 r_b = damper bar resistance.
 r_e = effective radius of damper bar end-rings.

 s = slip.
 s_o = slip at $\delta = 0$
 s_{av} = average slip.

 T_{do}' = open-circuit field time constant.
 T_d' = short-circuit field time constant.
 T_{do}'' = direct-axis open-circuit additional rotor circuits (considered as one circuit) time constant.
 T_d'' = direct-axis short-circuit additional rotor circuits (considered as one circuit) time constant.
 T_{qo}'' = quadrature-axis open-circuit additional rotor circuits (considered as one circuit) time constant.
 T_q'' = quadrature-axis short-circuit additional rotor circuits (considered as one circuit) time constant.
 T_a = armature circuit time constant.
 T_{d_u}' = unsaturated T_d' .
 T_{d_u}'' = unsaturated T_d'' .
 T_{q_u}'' = unsaturated T_q'' .
 T = torque.
 T_L = mechanical load torque.
 t = time.

 w_b = width of damper bars.
 w_r , w_{sr} = dimensions of damper bars, Fig. 7.8.

X_{ffd} = total field reactance.
 X_{nnd} (X_{11d} , X_{22d} , ...) = total reactance of additional rotor circuit in the direct-axis.
 X_{nnq} (X_{11q} , X_{22q} , ...) = total reactance of additional rotor circuit in the quadrature-axis.
 X_{afd} = mutual reactance between armature and field.
 X_{and} (X_{a1d} , X_{a2d} , ...) = mutual reactance between armature and additional rotor circuit in the direct-axis.
 X_{anq} (X_{a1q} , X_{a2q} , ...) = mutual reactance between armature and additional rotor circuit in the quadrature axis.
 X_{fnd} (X_{f1d} , X_{f2d} , ...) = mutual reactance between field and additional rotor circuit in the direct-axis.
 X_{nkd} = mutual reactance between n th and k th additional rotor circuits in the direct-axis.
 X_{nkq} = mutual reactance between n th and k th additional rotor circuits in the quadrature-axis.
 X_{ffd} = field reactance due to airgap flux.
 X_{fdd} = field reactance due to pole-tip and pole-body leakage.
 X_{gnd} = direct-axis damper bar reactance due to airgap flux.
 X_{gnq} = quadrature-axis damper bar reactance due to airgap flux.
 X_{bnd} = direct-axis damper bar reactance due to slot leakage.
 X_{bnq} = quadrature-axis damper bar reactance due to slot leakage.
 X_{end} = direct-axis damper bar reactance due to end-ring leakage.
 X_{ennq} = quadrature-axis damper bar reactance due to end-ring leakage.
 X_{ao} = unit impedance.

$$= \frac{e_{ao}}{i_{ao}}$$

x_a = armature reaction reactance of a non-salient-pole machine.
 x = synchronous reactance of a non-salient-pole machine.
 x_{ad} = direct-axis armature reaction reactance.
 x_{aq} = quadrature-axis armature reaction reactance.
 x_d = direct-axis synchronous reactance.
 x_q = quadrature-axis synchronous reactance.
 x_l = armature leakage reactance.
 x_{ld} = direct-axis airgap leakage reactance.
 x_{lq} = quadrature-axis airgap leakage reactance.
 x'_d = direct-axis transient reactance.
 x'_q = quadrature-axis transient reactance.
 x''_d = direct-axis sub-transient reactance.
 x''_q = quadrature-axis sub-transient reactance.
 x_2 = negative phase-sequence reactance.

$$= (x'_d + x''_q) / 2$$
 x_{s2} = negative phase-sequence reactance for the operating condition of line-to-line short circuit.

$$= \sqrt{x''_d x''_q}$$

x_0 = zero-phase-sequence reactance.
 x_e = external reactance in armature circuit.
 x'_d = static transient reactance in the direct-axis.
 x'_q = static transient reactance in the quadrature-axis.
 x''_d = static sub-transient reactance in the direct-axis.
 x''_q = static sub-transient reactance in the quadrature-axis.
 x_{dd} = direct-axis line-to-neutral static sub-transient reactance.
 x_{dq} = quadrature-axis line-to-neutral static sub-transient reactance.
 x_{sd} = direct-axis line-to-line static sub-transient reactance.
 x_{sq} = quadrature-axis line-to-line static sub-transient reactance.
 x'_{du} = unsaturated direct-axis transient reactance.
 x'_{qu} = unsaturated quadrature-axis transient reactance.
 x''_{du} = unsaturated direct-axis sub-transient reactance.
 x''_{qu} = unsaturated quadrature-axis sub-transient reactance.
 $x_d(p), x_q(p)$
 = operational impedances.

y_d = ratio of distance to direct-axis to half pole pitch.
 y_q = ratio of distance to quadrature-axis to half pole pitch.
 y_{nd} = y_d from n th damper bar.
 y_{nq} = y_q from n th damper bar.

Z_{ad} = direct-axis armature reaction impedances.
 Z_{aq} = quadrature-axis armature reaction impedance.
 Z_d = direct-axis impedance for forward current in starting.
 $= Z_{ad} + r_a + jx_l$
 Z_q = quadrature impedance for forward current in starting.
 $= Z_{aq} + r_a + jx_l$
 Z'_d = direct-axis impedance for backward current in starting.
 $= Z_{ad} + \frac{r_a}{2s-1} + jx_l$
 Z'_q = quadrature-axis impedance for backward current in starting.
 $= Z_{aq} + \frac{r_a}{2s-1} + jx_l$
 Z_d'' = direct-axis sub-transient impedance.
 Z_q'' = quadrature-axis sub-transient impedance.
 Z_1 = positive phase-sequence impedance.
 Z_2 = negative phase-sequence impedance.
 Z_0 = zero phase-sequence impedance.
 $Z_d(p), Z_q(p)$
 = operational impedances.

α = pole embrace.
 α_1 = $\alpha - 3.5pP_p'g$
 α = electrical space angle direct-axis leads the axis of the short-circuit phase at the instant of short circuit.
 α = electrical space angle measured from the direct-axis to any point on the periphery.

β = symbol for the function $-1 + \sqrt{1 + \frac{1}{4} \left(\frac{1 - \alpha}{P_0^{-1} g g_1} \right)^2}$

β = angle between axis of short circuit winding and axis of open phase winding.

γ = angle direct-axis leads the axis of the short-circuit phase at any instant.

γ = angle between current i and nominal open circuit voltage e_n .

γ = angle between any point P on rotor and the axis of phase A.
= $\Gamma + \theta$

δ = power angle.

$\Delta\delta$ = increment of power angle.

e = base of natural logarithms.

λ = specific permeance.

λ_i = specific permeance for the slot tooth-tip and zigzag leakage combined.

λ_e = specific permeance for the end-winding leakage.

λ_b = specific permeance for the belt leakage.

$\lambda_{i0}, \lambda_{e0}, \lambda_{b0}$ = specific permeances for zero phase-sequence current.

λ_p = specific permeance for the pole-body leakage.

λ_t = specific permeance for the pole-tip leakage.

Φ_d = fundamental direct-axis flux per pole produced by A_d .

Φ_q = fundamental quadrature-axis flux per pole produced by A_q .

Φ_a = resultant flux of Φ_d and Φ_q .

Φ_f = fundamental flux per pole produced by field.

Φ = resultant flux of Φ_a and Φ_f .

ϕ = terminal phase angle (P.F. angle).

Ψ_{fd} = field flux linkage.

Ψ_{fdo} = unit field flux linkage.

$\Psi_{nd} (\Psi_{1d}, \Psi_{2d}, \dots)$
= direct-axis additional rotor circuit flux linkage.

$\Psi_{nq} (\Psi_{1q}, \Psi_{2q}, \dots)$
= quadrature-axis additional rotor circuit flux linkage.

Ψ_{xdo} = unit additional rotor circuit flux linkage.

ψ_a, ψ_b, ψ_c
= instantaneous armature phase flux linkages.

$\psi_{ab}, \psi_{bc}, \psi_{ca}$
= armature flux linkages of two phases combined.

ψ_{ao} = unit armature flux linkage.

ψ_d = direct-axis flux linkage.

ψ_{dm} = magnitude of the vector form of ψ_d .

ψ_q = quadrature-axis flux linkage.

ψ_{qm} = magnitude of the vector form of ψ_q .

ψ_{da} = direct-axis flux linkage due to armature current.

ψ_{qa} = quadrature-axis flux linkage due to armature current.
 ψ_i = flux linkage due to armature current.
 ψ_m = flux linkage^{due} to the field.
 ψ'_m = flux linkage due to field just before short circuit.
~~= magnitude of vector~~
~~= magnitude of vector~~

ρ = ratio of maximum gap to minimum gap.
 ρ_o = ratio of peak value of open phase voltage after short circuit to that before short circuit.

$\Delta d_s'$ = field decrement for single-phase short circuit.
 $\Delta d_s''$ = damper winding decrement for single phase short circuit.
 $\Delta d'$ = field decrement for three-phase short circuit.
 $\Delta d''$ = damper winding decrement for three-phase short circuit.
 Δd_o = open-circuit field decrement.
 Δa = armature decrement.

θ = angle between direct-axis and the axis of phase A.
 θ_o = θ at $t = 0$

τ = pole pitch.
 τ_r = pole pitch on rotor diameter.
 τ_s = slot pitch.
 τ_b = pitch of damper bars.

ω = angular speed = $2\pi f$

Γ = angle between any point P on rotor and the direct axis.

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